9. LONGITUDINAL AND TRANSVERSE ELECTRIC WAVE PROPAGATION IN A MEDIUM

Abstract

An electric field of flux density D may be considered as an "elastic medium" having "density ρ" = $\mu_o D^2$ and exerting "pressure P" = D^2/ε_o, where μ_o is the permeability and ε_o the permittivity of vacuum. Longitudinal waves, excited by oscillations of charged particles in a medium, are propagated at the speed of light $c = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{1}{\mu_o \varepsilon_o}}$. The longitudinal waves, between the charged particles, are absorbed by the oscillating particles. Transverse waves, set up by the oscillating particles, are emitted as electromagnetic radiation, propagated at the speed of light c .

Keywords: density, light, polarization, pressure, velocity, waves

9.1 Introduction

A wave is a process by which an influence reaches out from one body to another without transfer of matter, mass or electric charge. This aspect of wave motion was elucidated by C.G. Darwin (*New Conceptions of Matter*, 1931, p. 29) [1], Professor of Physics at the University of Edinburgh, who wrote:

> *"The most elementary way in which I can attract anyone's attention is to throw a stone at him. Another is to poke him with a stick, without transfer of matter from me to him - a small motion that I produce in the stick at my end, turns into a small motion at his end".*

A small motion at one end of a medium producing a small motion at the other end, without transfer of matter, takes time to travel in or along the medium. The small motion is transmitted as a succession of pulses or a wave of displacements or compressions and rarefactions along the length of the medium which may be solid, liquid, gas or an electric field. The speed of transmission depends on some physical properties, such as elasticity and density of the medium.

The small displacements, oscillations or vibrations in a wave motion may be in the direction of propagation, in which case we have a longitudinal wave, like vibrations in a rigid rod or like sound waves in an air column. Where the displacements are perpendicular to the direction of propagation, we get a transverse wave, like waves in a stretched string, waves on the surface of water or electromagnetic waves.

James Clerk Maxwell (1865) [2], in his epoch-making treatise, showed that waves from oscillating electric currents consisted of transverse vibrations of electric and magnetic fields propagated at the speed of light c, hence the term electromagnetic waves. He concluded that electromagnetic radiation is a vast spectrum from radio frequencies at the lower end, through microwave, infrared light, visible light and ultraviolet light to x-rays and gamma rays at the higher end, all propagated at speed $c = \sqrt{\dfrac{1}{\mu_o \varepsilon_o}} = 2.998 \times 10^8\ m/s$) in free space or vacuum, where μ_o is the permeability and ε_o the permittivity of a vacuum. .

The spectrum of visible (white) light range from wavelength (red) $7 \times 10^{-7}\ m$ to (violet) $4 \times 10^{-7}\ m$. Found below the visible spectrum are the infrared rays ($10^{-6}\ m$ to $10^{-3}\ m$). Beyond the visible region are the ultraviolet radiation ($1.5 \times 10^{-8}\ m$ to $10^{-7}\ m$), the x-rays and the gamma rays. The light radiations are produced from the motions of charged particles in the electric fields [3, 4] of the atoms of a material.

This paper considers electric field as an "elastic medium" having "density ρ" and exerting "pressure P". As such an electric field supports the propagation of a longitudinal wave with the speed of light $c = \sqrt{\dfrac{1}{\mu_o \varepsilon_o}} = \sqrt{\dfrac{P}{\rho}}$. Such a wave may be excited by the motion of electric charges in an electric field. A longitudinal wave, which exists between the electric charges in a medium, is not, in any way, polarized.

A transverse wave, on the other hand, is always polarized. Transverse electromagnetic waves are quickly attenuated in a conducting medium. The attenuation is due to a phenomenon called Skin Effect [5, 6, 7].

In a transverse wave there are oscillations of the electric and magnetic fields perpendicular to the direction of propagation. The electric vector determines the polarization. Where the transverse oscillations of the electric fields are more in one plane than the other, the wave is said to be polarized. If the oscillations are random, the wave is unpolarized.

9.2 “Pressure” in an electric field

Figure 9.1 shows an electric field of intensity ***E*** cutting normally through an element of (shaded) surface area *(δ**A**)* at a point *R* distance (*x + l)* from an origin. The quantity of electric charge *(δq)* encompassed by the (shaded) area *(δ**A**)* is given by Gauss’s law as the scalar product:

$$(\delta q) = \varepsilon_o \mathbf{E}.(\delta \mathbf{A}) = \varepsilon_o E(\delta A) \tag{9.1}$$

where ε_o is the permittivity of free space. The force *(δ**F**)* exerted over the surface area *(δ**A**)* is:

$$(\delta \mathbf{F}) = \varepsilon_o E(\delta A)\mathbf{E} = \varepsilon_o E^2 (\delta A)\hat{\mathbf{u}} \tag{9.2}$$

where $\hat{\boldsymbol{u}}$ is a unit vector in the direction of the electric field as well as the area. Equation (9.2) gives the force per unit area, equal to the pressure *P* or stress at *R*, as:

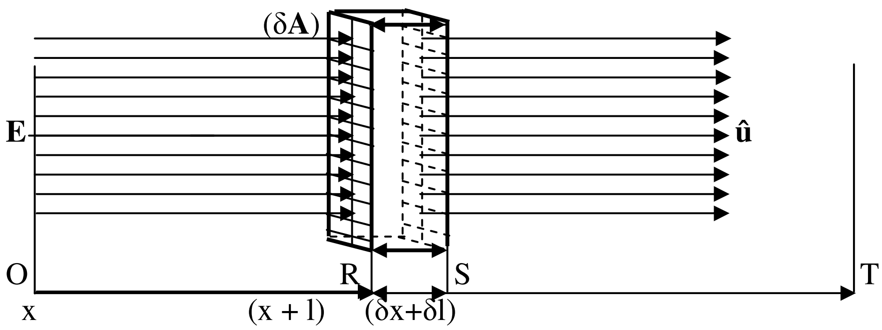

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Figure 9.1. Electric field ***E*** across an element of surface area *(δ**A**)* at R

$$P = \varepsilon_o E^2 = \frac{D^2}{\varepsilon_o} \tag{9.3}$$

where $D = \varepsilon_o E_o$ is the electric flux density.

9.3 “Elasticity” of an electrostatic field

An electric field may be stressed and strained, like a column of air or an elastic rod. Thus an electric field is “elastic”. So, a stream of compressions and rarefactions may propagate, at the speed of light *c*, along an electrostatic field, to constitute a longitudinal electric wave.

In Figure 9.1, if a segment of an electric field, of intensity ***E*** and length *(δx)* across surface area *(δ**A**)* at *R*, suffers an elongation (rarefaction) or shortening (compression) of length *(δl)*, the strain is *(δl)/(δx)*. The total force ***f*** across the (shaded) surface area *(δ**A**)* at *R*, as a result of the electrostatic force (equation 9.2) and the strain in addition, is proposed, with *P* given by equation (3), as:

$$\mathbf{f} = \varepsilon_o E^2 (\delta \mathbf{A}) \left(1 + \frac{\delta l}{\delta x}\right) = P(\delta \mathbf{A}) \left(1 + \frac{\delta l}{\delta x}\right) \tag{9.4}$$

The validity of equation (9.4) lies on its leading to the correct result in order to arrive at the wave equation and derive an expression for the speed of a longitudinal wave in an electrostatic field.

9.4 "Density" of an electric field

In Figure 9.1, the potential *V* at *R* is given by the integral:

$$V = \int \mathbf{E}.(d\mathbf{x}) = \int E(dx)$$

If the electric field at *R* is increased by a small amount *(δ**E**)*. The corresponding increase in the electric charge is $\varepsilon_o(\delta\mathbf{E}).(\delta\mathbf{A})$. Work done in increasing the charge, by infinitesimal amounts, is:

$$w = \iiint \varepsilon_o (d\mathbf{E}).(d\mathbf{A}) E(dx) = \iint \varepsilon_o E(dE)(d\tau) \tag{9.5}$$

where *(δτ)* is an element of volume. Integrating equation (9.5) gives:

$$w = \iint \varepsilon_o E(dE)(d\tau) = \frac{1}{2}\int \varepsilon_o E^2 (d\tau) \tag{9.6}$$

The energy per unit volume υ, is:

$$\upsilon = \frac{1}{2}\varepsilon_o E^2 = \frac{D^2}{2\varepsilon_o} \tag{9.7}$$

Equation (9.7), for energy per unit volume, is not the same as (9.3).

The author [8] showed that the energy content of a mass *m* at rest, is

$$w = \frac{m}{2\mu_o \varepsilon_o} = \frac{1}{2}mc^2 \tag{9.8}$$

where μ_o is the permeability and ε_o the permittivity free space (vacuum). If "ρ" is the "density" of the electric field, the energy per unit volume, is:

$$\upsilon = \frac{D^2}{2\varepsilon_o} = \frac{\rho}{2\mu_o \varepsilon_o}$$

$$\rho = \mu_o D^2 \qquad (9.9)$$

Speed of wave propagation along the electric field is obtained as:

$$c = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{D^2}{\mu_o \varepsilon_o D^2}} = \sqrt{\frac{1}{\mu_o \varepsilon_o}} \qquad (9.10)$$

9.5 Longitudinal wave equation

Figure 9.1 shows an element of (shaded) surface area *(δ**A**)* at a point *R* distance *(x +* 1*)* from an origin. A segment of the electric field with surface area *(δ**A**)* and initial width *(δ**x**)* oscillates between two points *O* and *T*, through a displacement *l*, with speed *(dl/dt)* at time *t*.

At the point *O*, the segment has original volume *(δA)(δx)*. On moving from *O* through a distance *l*, with speed *(dl/dt)*, let the segment increase in width from the initial *(δx)* to *(δx + δl)* so that the volume increases by *(δA)(δl)*. The segment, of mass *ρ(δA)(δx)*, oscillates with speed *(dl/dt)* and acceleration (d^2l/dt^2) such that the accelerating force on the mass is equal to the impressed force on the segment *RS* of original volume *(δA)(δx)*.

The segment *RS*, in getting its width increased from *(δx)* to *(δx+δl)*, suffers a strain equal to *(δl/δx)*. The force ***f*** on (shaded) surface area *(δ**A**)*, with stress *P*, at *R*, is given by equation (9.4) as:

$$\mathbf{f} = P(\delta \mathbf{A})\left(1 + \frac{\delta l}{\delta x}\right) \qquad (9.11)$$

The difference in force between the (shaded) surfaces at *R* and *S*, within a distance *(δx)*, is:

$$\frac{d\mathbf{f}}{dx}(\delta x) = P(\delta \mathbf{A})\left(\frac{d^2 l}{dx^2}\right)(\delta x) \qquad (9.12)$$

Equation (9.12) gives the (differential) impressed force on the segment *RS*, which imparts acceleration (d^2l/dt^2) on mass *ρ(δA)(δx)*. Newton's second law of motion gives the equation:

$$P(\delta \mathbf{A})\left(\frac{d^2 l}{dx^2}\right)(\delta x) = \rho(\delta \mathbf{A})(\delta x)\left(\frac{d^2 l}{dt^2}\right)$$

$$\left(\frac{d^2 l}{dx^2}\right) = \frac{\rho}{P}\left(\frac{d^2 l}{dt^2}\right) \qquad (9.13)$$

$$\left(\frac{d^2 l}{dx^2}\right) = \frac{\rho}{P}\left(\frac{d^2 l}{dt^2}\right) = \mu_o \varepsilon_o \left(\frac{d^2 l}{dt^2}\right) = \frac{1}{c^2}\left(\frac{d^2 l}{dt^2}\right) \tag{9.14}$$

This is the wave equation, for a longitudinal wave with displacement *l* at time t, propagated with speed *c* in the **x**-direction.

9.6 Transverse wave equation

Consider an electric charge of magnitude *Q* oscillating in the vertical direction due to an external electric fields es in a body, as depicted below:

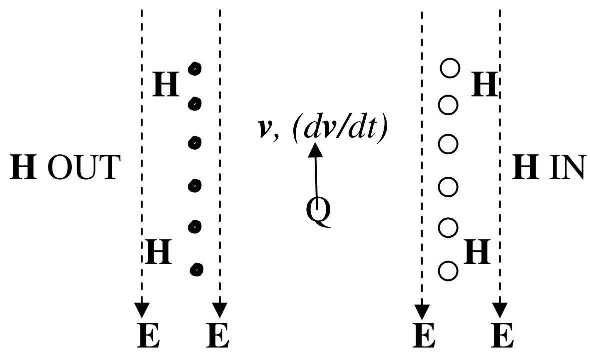

Figure 9.2 An electric charge *Q* moving with velocity ***v*** and acceleration *(dv/dt)* at time *t* producing a magnetic field of intensity ***H*** and electrodynamic field of intensity ***E***

The charge *Q* moving with velocity ***v*** (in the **z**-direction) and acceleration *(dv/dt)* at time *t*, creates a magnetic field of intensity ***H*** (in the **y**-direction) and an electrodynamic field of intensity ***E*** (in the –**z** direction) as shown in Figure 9.2. The dynamic configuration gives rise to an electromagnetic wave propagated in (the **x**-direction) a direction normal to the surface (out) of the page.

A changing magnetic field ***H*** induces a voltage by making an electric field ***E***, according to Faraday's law [9] as given by Maxwell's equation:

$$\nabla \times \mathbf{E} = -\mu_o \frac{\partial \mathbf{H}}{\partial t} \tag{9.15}$$

where $\nabla\times$ denotes the curl of a vector and μ_o is permeability of vacuum.

In free space (vacuum) and in the absence of any conduction current, Ampere's law gives the Maxwell equation [10]:

$$\nabla \times \mathbf{H} = \varepsilon_o \frac{\partial \mathbf{E}}{\partial t} \tag{9.16}$$

whee ε_o is the permittivity of vacuum.

Taking the curl of equation (9.15) gives:

$$\nabla \times \nabla \times \mathbf{E} = -\mu_o \frac{\partial}{\partial t} \nabla \times \mathbf{H}$$

$$\nabla \nabla . \mathbf{E} - \nabla^2 \mathbf{E} = -\mu_o \varepsilon_o \frac{\partial}{\partial t} \frac{\partial \mathbf{E}}{\partial t}$$

where **∇.** denotes the divergence of a vector. Since there is no distribution of charges, **∇.E** is zero and we obtain:

$$\nabla^2 \mathbf{E} = \mu_o \varepsilon_o \frac{\partial^2 \mathbf{E}}{\partial t^2} \tag{9.17}$$

$$\frac{\partial^2 E_z}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 E_z}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2} \tag{9.18}$$

This is the one-dimensional transverse wave equation with electric field E_z at time t, propagated with speed c in the **x**-direction.

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9.7 Skin Effect

A transverse electromagnetic wave is polarized with the electric and magnetic fields orthogonal to one another and perpendicular to the direction of propagation. An electromagnetic wave, normally incident on the surface of a medium, has the electric and magnetic fields lying on the surface. The wave may be reflected, absorbed or transmitted depending on the nature of the medium. For a conducting medium the wave is attenuated and absorbed within a short distance of penetration due to a phenomenon called Skin Effect. The attenuation is a result of induced current flow and dissipation of energy in the medium. The Skin Thickness or Skin Depth is the distance of penetration within which the current density is reduced to *1/e* (about *1/2.783*) of its magnitude at the surface.

Salty sea water, a poor conductor of electricity, has conductivity of about $5\ (ohm\text{-}m)^{-1}$. The Skin Thickness for an electromagnetic wave of radio frequency $3 \times 10^7\ Hz$ (wavelength *10 m*), falling on sea water, is $4.1 \times 10^{-2}\ m$. Thus radio communication should not be possible between two aerials, some metres apart, submerged in the sea. For yellow light of wavelength $6 \times 10^{-7}\ m$ (frequency $5 \times 10^{14}\ Hz$), in sea water, the Skin Thickness might be $10^{-5}\ m$, a very thin thickness indeed. So, sea water should have been opaque to light radiation, but it is not. Is there any difference between light and radio waves as electromagnetic radiations?

9.8 Polarization of light

Figure 9.3 depicts a ray of polarized light *PO* from a source at a point *P*, incident on a plane surface *AB* of a medium. *ON* is the normal at the point of incidence *O* and ι is the angle of incidence. *OQ* is the reflected ray at angle ρ to the normal. *OR* is the refracted ray and τ is the angle of refraction. Transverse electric field oscillations, in the plane of incidence (surface of the paper) are indicated as double arrows and oscillations perpendicular to the plane of incidence are shown as small circles.

The law of reflection makes $\rho = \iota$ and Snell's law gives:

$$\frac{\sin \iota}{\sin \tau} = \mu \tag{9.19}$$

At an angle of incidence, other than 0 or π/2 radians, less of vibrations in the plane of incidence are reflected, while components perpendicular to the plane are not affected. So, reflected light is polarized perpendicular to the plane of incidence. At a particular angle of incidence β, called *Brewster angle* or *Polarization angle*, the reflection of components in the plane of incidence is zero. At this angle ($\iota = \beta = \rho$), transverse vibrations (in the plane of incidence), in the refracted ray, should be in the direction of the reflected ray; the reflected ray being perpendicular to the refracted ray, making $(\tau + \beta) = \pi/2$ radians, to give the equation:

$$\frac{\sin \beta}{\sin \tau} = \frac{\sin \beta}{\sin\left(\frac{\pi}{2} - \beta\right)} = \frac{\sin \beta}{\cos \beta} = \tan \beta = \mu \tag{9.20}$$

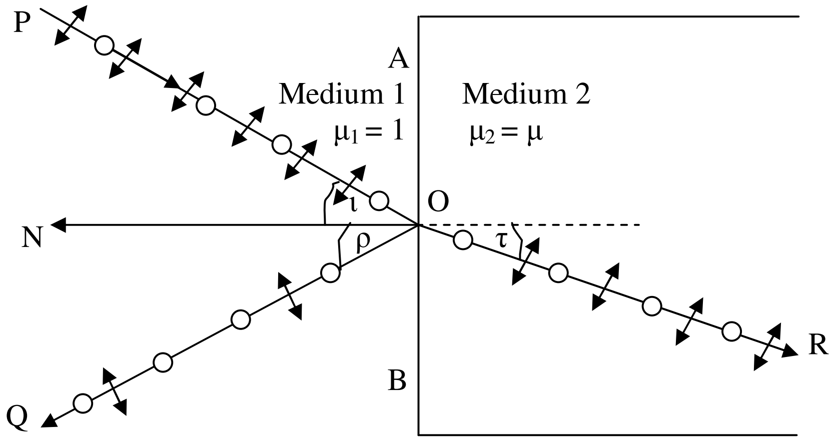

Figure 9.3 A ray of light *PO* reflected from a plane surface *AB*, with *OQ* as the reflected ray and *OR* the refracted ray. The transverse oscillations in the plane of incidence (surface of the paper) are shown as double arrows and oscillations perpendicular to the plane of incidence as small circles along the ray.

Fresnel's equations [11] give the ratio of intensities of each of the two polarization components of light which is reflected or refracted between two media with different indices of refraction, in terms of the angle of incidence and angle of refraction..

9.9 Conclusion

An electric field may be considered as an elastic medium which can be stressed and strained in relation to moving charged particles. As such, there could be propagation of longitudinal waves, at the speed of light, between the charged particles. The longitudinal oscillations are absorbed by the atomic particles, with a manifestation of heat,

Motions of electric charges also give rise to transverse waves with oscillations perpendicular to the direction of propagation. The transverse are emitted and propagated in space as light radiation.

The fact that fish can be seen swimming in clear sea water and divers use search lights to find their way in the depth of the sea indicates that light waves are transmitted in sea water, whereas radio waves are readily attenuated even in a poor conductor like salty sea water. Perhaps, the difference in polarization may explain why radio waves are quickly attenuated in sea water while light waves may be transmitted.

9.10 References

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