8. EXPLANATIONS OF THE RESULTS OF ROGER'S AND BERTOZZI'S EXPERIMENTS WITHOUT RESORT TO SPECIAL RELATIVITY

Abstract

The results of Roger's experiment with electrons revolving at defined speeds in circular orbits and Bertozzi's experiment with high-speed electrons moving in a linear accelerator, can be explained as due to accelerating force on an electron decreasing with speed, reducing to zero at the speed of light. This is in contrast to special relativity where the results are ascribed to the mass of a moving electron increasing with speed, becoming infinitely large at the speed of light. The result of Roger's experiment is also shown to be in agreement with *radiational electrodynamics* for circular revolution of electrons round a centre of force. The explanations lead to the speed light as the ultimate speed without infinite mass.

Keywords: Rectilinear and circular motion, special relativity, speed.

8.1 Introduction

In classical electrodynamics [1, 2], the mass of a particle is independent of its speed and a charged particle, such as an electron, can be accelerated by an electrostatic force, beyond the speed of light. But observations on accelerated electrons, the lightest particles known in nature, showed that their speeds could not exceed that of light. Relativistic electrodynamics [3, 4] and *radiational electrodynamics* [5] deal with the issues that restrain accelerated particles from going beyond the speed of light.

Relativistic electrodynamics explaines the speed of light being a limit by positing that the mass m of a moving particle increases with its speed v, becoming infinitely large at the speed of light c. The mass-velocity formula, of special relativity, is:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_o \qquad (8.1)$$

where m_o is the rest mass. This position is apparently plausible as an infinite mass cannot be pushed any faster by any finite force.

In equation (8.1), the difficulty with infinite masses, at the speed of light $(v = c)$, is avoided by insisting that the speed v may be as near as possible, but it never really becomes equal to c. Photons, as "particles" supposed to move at the speed of light, are given zero rest mass. Also, the speed of light c in a vacuum is made an absolute constant, independent of the speed of the source of light or the speed of the observer.

Radiational electrodynamics proposes that the speed of light c is an ultimate limit because the accelerating force exerted by an eletrostatic field, on a moving charged particle, decreases with the speed of the particle. The accelerating force reduces to zero at the speed of light and the particle continues to move at that speed, with the rest mass m_o, in accordance with Newton's second law of motion.

In *radiational electrodynamics*, decrease of accelerating force with speed, in the revolution of an electron round a centre of force, gives the same effect as apparent increase in mass with speed in accordance with equation (8.1). Therefore, Roger's experiment (1939) [6], supposed to have proved increase of mass with speed, might as well have confirmed decrease of accelerating force with speed, as far as circular motion round a central force is concerned.

In rectilinear motion, it was found that electrons cannot be accelerated beyond the speed of light, no matter the magnitude of the accelerating potential in a linear accelerator. The existence of a limiting speed, equal to the speed of light, was clearly demonstrated in Bertozzi's experiment (1964) [7]. Here, again, the limiting speed was attributed to mass increasing with speed, becoming infinitely large at the speed of light, as per the relativistic equation (8.1). An accelerating force decreasing with speed, becoming zero at the speed of light, in accordance with *radiational electrodynamics*, should also lead to that speed being an ultimate limit, in accordance with Newton's laws of motion.

In this paper, the motion of electrons, in an electrostatic field, is treated under classical, relativistic and *radiational electrodynamics*. It is found that the results of Roger's and Bertozzi's experiments are in agreement with predictions of *radiational electrodynamics*, but not on the basis of mass increasing to become infinitely large at the speed of light. Under *radiational electrodynamics*, it is shown that the speed of light is a limit not because mass becomes infinitely large at that speed but as a result of accelerating force exerted by an electrostatatic field, on a moving electron, decreasing with speed, reducing to zero at the speed of light.

8.2 Classical electrodynamics

8.2.1 Potential energy lost by an accelerated electron

In classical electrodynamics, the accelerating force ***F*** on an electron of charge *–e* and constant mass *m*, moving with velocity ***v*** and acceleration *dv/dt* at time *t*, in an electrostatic field of intensity ***E***, is given, in accordance with Newton's second law of motion, by the vector equation:

$$\mathbf{F} = -e\mathbf{E} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} \tag{8.2}$$

For rectilinear motion, in the direction of a displacement ***x***, equation (8.2), with *E* as the magnitude of ***E***, becomes:

$$-eE\hat{\mathbf{u}} = -m\frac{dv}{dt}\hat{\mathbf{u}} = -mv\frac{dv}{dx}\hat{\mathbf{u}} \tag{8.3}$$

where ***û*** is a unit vector in the direction of the electrostatic field ***E*** and the displacement ***x***. The scalar equation is:

$$eE = mv\frac{dv}{dx} \tag{8.4}$$

The potential energy *P* lost by the moving electron or work done on the electron, in being accelerated with constant mass *m*, through a distance *x*, to a speed *v* from rest, is gven by the definite integral:

$$P = \int_0^x eE(dx) = m\int_0^v v(dv) \tag{8.5}$$

Integrating, equation (8.5) becomes:

$$P = \frac{1}{2}mv^2 = \frac{1}{2}m_o v^2$$

$$\frac{P}{m_o c^2} = \frac{1}{2}\left(\frac{v}{c}\right)^2 \tag{8.6}$$

Here, the potential energy *P* lost is equal to the kinetic energy gained, as there is no consideration of energy radiation.

8.2.2 Potential energy gained by a deccelerated electron

In classical electrodynamics, an electron, moving at the speed of light *c*, can be decelerated to a stop and may be accelerated in the opposite direction to reach a speed greater than **–***c*. The potential energy *P* gained in decelerating an electron from the speed of light *c* to a speed *v*, is:

$$P = \frac{1}{2} m(c^2 - v^2) \qquad (8.7)$$

8.2.3 Circular revolution of an electron

Figure 8.1 shows an electron of mass *m* and charge *–e* revolving in a circle of radius *r* in an electrostatic field of intensity ***E*** due to a point charge *Q* at the centre *O*. The accelerating force ***F***, in accordance with Newton' second law of motion, is given by the vector equation:

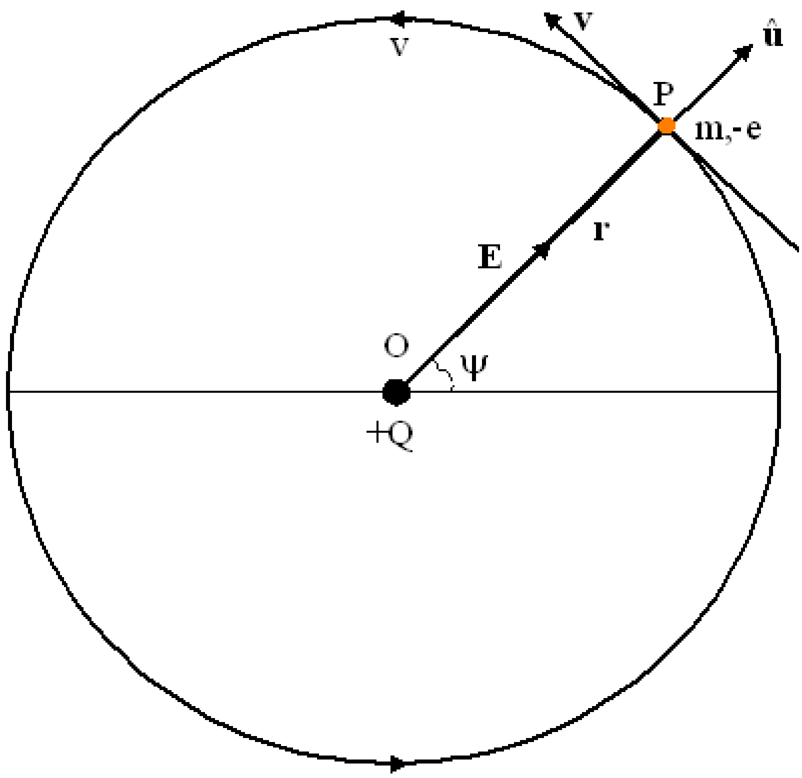

Figure 8.1. An electron of mass *m* and charge *–e* revolving with speed *v* in a circle of radius *r*, in a radial field of intensity ***E*** due to a positive charge *Q* at the centre *O*.

$$\mathbf{F} = -e\mathbf{E} = m\frac{d\mathbf{v}}{dt} = -m\frac{v^2}{r}\hat{\mathbf{u}} \tag{8.8}$$

where $(-v^2/r)\hat{\mathbf{u}}$ is the centripetal acceleration due to the accelerating force. The scalar equation is:

$$eE = m\frac{v^2}{r} \tag{8.9}$$

$$\frac{eEr}{m_o v^2} = \frac{m}{m_o} \tag{8.10}$$

where m_o is the rest mass. In classical electrodynamics, $m = m_o$ is a constant and equation (8.10) should give a unity, for all values of E, r and v. This is not what was observed in laboratory experiments.

8.3 Relativistic electrodynamics

8.3.1 Potential energy lost by an accelerated electron

In relativistic electrodynamics, the kinetic energy K gained by an electron or the work done, in being accelerated to a speed v from rest, is the potential energy P lost. The kinetic energy K of a particle of mass m and rest mass m_o moving with speed v, is given by the relativistic equation:

$$K = P = mc^2 - m_o c^2$$

$$P = \frac{m_o c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_o c^2$$

$$\frac{P}{m_o c^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \tag{8.11}$$

where m_o is the rest mass (at $v = 0$) and c the speed of light in a vacuum. Bertozzi's experiment was conducted to verify equation (8.11) and it appeared to have done so in a remarkable way.

8.3.2 Potential energy gained by a deccelerated electron

In relativistic electrodynamics, an electron moving at the speed of light *c* (with infinite mass), cannot be stopped by any decelerating force. The electron continues to move at the same speed of light *c*, gaining potential energy without losing kineticl energy, contrary to the principle of conservation of energy.

8.3.3 Circular revolution of an electron

In relativistic electrodynamics, the accelerating force ***F*** on an electron is independent of its velocity ***v*** at time *t* in an electrostatic field of intensity ***E***, but the mass *m* increases with speed *v* in accordance with the mass-velocity formula, equation (8.1). In Figure 8.1, constant centripetal acceleration $(-v^2/r)\hat{\mathbf{u}}$, gives the force ***F***, in accordance with Newton's second law of motion, as the vector:

$$\mathbf{F} = -e\mathbf{E} = -m\frac{v^2}{r}\hat{\mathbf{u}} \qquad (8.12)$$

Combining equation (8.12) with equation (8.1), gives:

$$eE = m\frac{v^2}{r} = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}\frac{v^2}{r} \qquad (8.13)$$

$$\frac{eEr}{m_o v^2} = \frac{m}{m_o} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (8.14)$$

Roger's experiment set out to verify equation (8.14) and it did so convincingly. It provided an evidence of apparent increase in mass with speed in accordance with special relativity.

8.4 *Radiational electrodynamics*

8.4.1 Motion of an electron in an electrostatic field

Figure 8.2 depicts an electron of charge $-e$ and constant mass $m = m_o$, moving at a point *P* with velocity ***v*** at time t, in an electrostatic field of intensity ***E*** due to a stationary source charge $+Q$ at the origin *O*. The velocity ***v*** is at an angle θ to the accelerating force ***F***, which is a force of attraction in the **PO** direction. The relative velocity between the accelerating force (propagated with velocity of light ***c***) and the electron moving with velocity **v**, is the vector (***c*** – ***v***). The velocity of light ***c***, is inclined at the aberration angle α to the accelerating force ***F***, such that:

$$\sin\alpha = \frac{v}{c}\sin\theta \tag{8.15}$$

where *v* and *c* are the magnitudes of ***v*** and ***c*** respectively.

In *radiational electrodynamics*, the accelerating force ***F***, with reference to Fig. 8.2, is given by the vector equation:

$$\mathbf{F} = \frac{eE}{c}(\mathbf{c} - \mathbf{v}) = m\frac{d\mathbf{v}}{dt} \tag{8.16}$$

where *E* is the magnitude of the electrostatic field of intensity ***E***.

Expanding equation (8.16) by taking the *modulus* of (***c*** – ***v***), with respect to the angles θ and α in Figure 8.2, gives the equation:

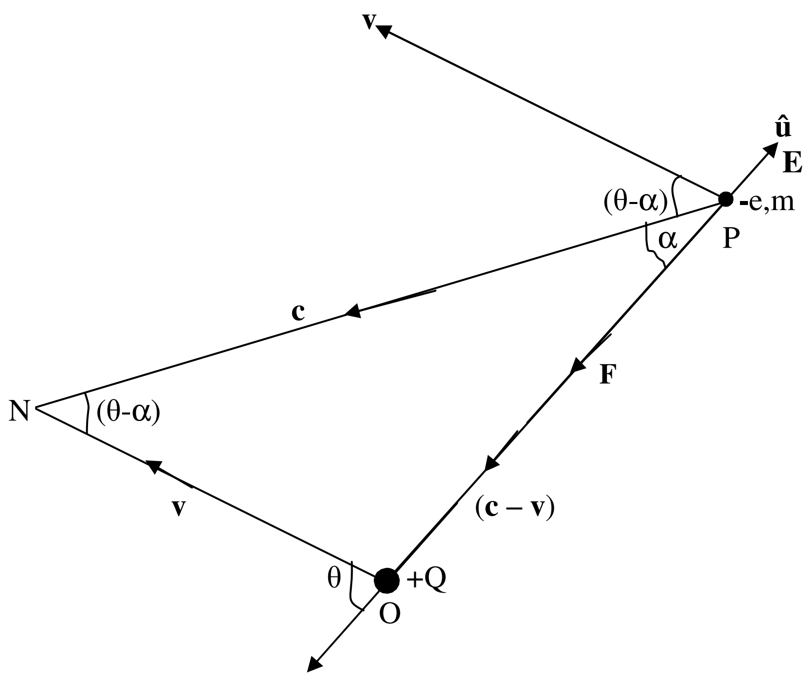

Figure 8.2. An electron of charge –*e* and mass *m* moving, at a point *P*, with velocity ***v***, at an angle θ to the accelerating force ***F***. The unit vector $\hat{\boldsymbol{u}}$ is in the direction of the electrostatic field **E** due to a positive charge Q at O.

$$\mathbf{F} = \frac{-eE}{c}\sqrt{c^2 + v^2 - 2cv\{\cos(\theta - \alpha)\}}\hat{\mathbf{u}} = m\frac{d\mathbf{v}}{dt} \qquad (8.17)$$

where $(\theta - \alpha)$ is the angle between $\boldsymbol{c}$ and $\boldsymbol{v}$ and $\hat{\mathbf{u}}$ is a unit vector in the direction of the field **E,** opposite to the direction of $(\boldsymbol{c} - \boldsymbol{v})$. The electron can move in a straight line, in the direction of the force, with acceleration where $\theta = 0$ or against the force with deceleration where $\theta = \pi$ radians or it can revolve in a circle, with constant speed v, if θ is equal to $\pi/2$ radians. Motion in a circle is at right angle to a radial electric field without change in potential or kinetic energy,

8.4.2 Potential energy lost by an accelerated electron

For an accelerated electron, equations (8.15) and (8.17), with $\theta = 0$, give the vector equation:

$$\mathbf{F} = -eE\left(1 - \frac{v}{c}\right)\hat{\mathbf{u}} = -m\frac{dv}{dt}\hat{\mathbf{u}} \qquad (8.18)$$

The scalar equation is:

$$eE\left(1 - \frac{v}{c}\right) = m\frac{dv}{dt} = mv\frac{dv}{dx} \qquad (8.19)$$

The potential energy P lost in accelerating the electron, through a distance x, to a speed v from rest, is given by the integral:

$$P = \int_0^x eE(dx) = \int_0^v mv\frac{dv}{1 - \frac{v}{c}} \qquad (8.20)$$

Resolving the right-hand integral into partial fractions, we obtain:

$$P = mc\int_0^v \left(\frac{1}{1 - \frac{v}{c}} - 1\right)dv \qquad (8.21)$$

$$P = -mc^2 \ln\left(1 - \frac{v}{c}\right) - mcv \qquad (8.22)$$

$$\frac{p}{mc^2} = -\ln\left(1 - \frac{v}{c}\right) - \frac{v}{c} \qquad (8.23)$$

Equation (8.23) for *radiational electrodynamics*, should be compared with equation (8.11) for relativistic electrodynamics and equation (8.6) for classical electrodynamics.

8.4.3 Potential energy gained by a decelerated electron

For a decelerated electron, equations (8.15) and (8.17), with $\theta = \pi$ radians, give the vector equation:

$$\mathbf{F} = -eE\left(1 + \frac{v}{c}\right)\hat{\mathbf{u}} = m\frac{dv}{dt}\hat{\mathbf{u}} \qquad (8.24)$$

$$eE\left(1 + \frac{v}{c}\right) = -m\frac{dv}{dt} = -mv\frac{dv}{dx} \qquad (8.25)$$

Potential energy P gained in deceleration from the speed of light c to v, is:

$$P = \int_0^x eE(dx) = \int_c^v -mv\frac{dv}{1 + \frac{v}{c}} \qquad (8.26)$$

$$P = -mc\int_c^v \left(1 - \frac{1}{1 + \frac{v}{c}}\right)dv \qquad (8.27)$$

$$P = mc^2 \ln\frac{1}{2}\left(1 + \frac{v}{c}\right) + mc^2\left(1 - \frac{v}{c}\right) \qquad (8.28)$$

8.4.4 Accelerating force in circular revolution

With the angle $\theta = \pi/2$ radians, the electron revolves in a circle of radius r with constant speed v and centripetal acceleration $(-v^2/r)\hat{\mathbf{u}}$. Noting that $\cos(\pi/2 - \alpha) = \sin\alpha$, equations (8.15) and (8.17) give:

$$\mathbf{F} = -eE\sqrt{1 - \frac{v^2}{c^2}}\hat{\mathbf{u}} = -m\frac{v^2}{r}\hat{\mathbf{u}} \qquad (8.29)$$

$$eE = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}\frac{v^2}{r} = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}\frac{v^2}{r} \qquad (8.30)$$

Equation (8.30), with $m = m_o$, is identical to (8.13), as confirmed by Roger's experiment performed in 1939.

8.5 Roger's experiment

An experiment by M. Roger [6], seem to support the relativistic mass-velocity formula. In this experiment, an electron of mass *m*, moving at a well defined speed *v*, was made to enter a radial electrostatic field. The electron was deflected to move in a circle of radius *r* under a centripetal accelerating force of magnitude *F*, to give equation (8.14) for m/m_o. The results of Roger's experiment are shown in Table 8.1.

TABLE 8.1. RESULTS OF ROGER'S EXPERIMENT FOR THREE DIFFERENT SPEEDS
($c = 2.998 \times 10^8$ m/sec, $e/m_o = 1.759 \times 10^{11}$ C/kg)

SPEED v m/sec	v/c	Er $\times 10^5$ V	OBSERVED m/m_o	CALCULATED $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$
1.900×10^8	0.634	2.671	1.302	1.293
2.087×10^8	0.696	3.487	1.408	1.393
2.247×10^8	0.750	4.341	1.512	1.511
2.998×10^8	1.000	?	?	∞

The observed values of the ratio m/m_o, obtained for three different speeds, by measuring *Er*, and the calculated values from equation (8.14), were in close agreement, as shown in Table 8.1. Roger's experiment has seemingly verified the theory of special relativity, which predicted that the mass *m* of an electron increases with its speed *v*, becoming infinitely large at the speed of light *c*. However, the speed, in equation (8.1) is never allowed to reach that of light. But Bertozzi's experiment showed that electrons are easily accelerated to the speed of light through a potential energy of 15 MeV or over.

8.6 Bertozzi's experiment

A remarkable demonstration of the speed of light being a universal limiting speed, was in an experiment conducted by William Bertozzi, at the Massachusetts Institute of Technology in 1964 [7]. In this experiment, the speed *v* of high-energy electrons was determined by measuring the time *T* required for them to traverse a distance of 8.4 metres after having been accelerated through a potential energy *P* inside a linear accelerator. Bertozzi's experimental data is reproduced in Table 8.2. It was clearly demonstrated, in this experiment, that electrons accelerated through energies of 15 MeV or more, attain, for all practical purposes, the speed of light *c* as a limit.

TABLE 8.2 RESULTS OF BERTOZZI'S EXPERIMENTS WITH ELECTRONS ACCELERATED THROUGH ENERGY P
(m_oc^2 = 0.5 MeV, v = 8.4/T m/sec)

P MeV	P/m_oc^2	T x 10^{-8} sec.	v x 10^8 m/sec	v/c	$(v/c)^2$
0.5	1	3.23	2.60	0.87	0.76
1.0	2	308	2.73	0.91	0.88
1.5	3	2.92	2.88	0.96	092
4.5	9	2.84	2.96	0.99	0.97
15.0	30	2.80	3.00	1.00	1.00

A graph of P/m_oc^2 (potential energy in units of m_oc^2) against $(v/c)^2$ (speed squared in units c^2), is shown in Figure 8.3; the solid line (A) in accordance with classical electrodynamics (equation 8.6), the dashed curve (B) according to relativistic electrodynamics (equation 8.11) and the dashed curve (C) according to radiational electrodynamics (equation 8.23). The solid squares are the results of Bertozzi's experiment (Table 8.2). Bertozzi' experiment appears to be in agreement with relativistic and radiational electrodynamics but away from classical electrodynamics.

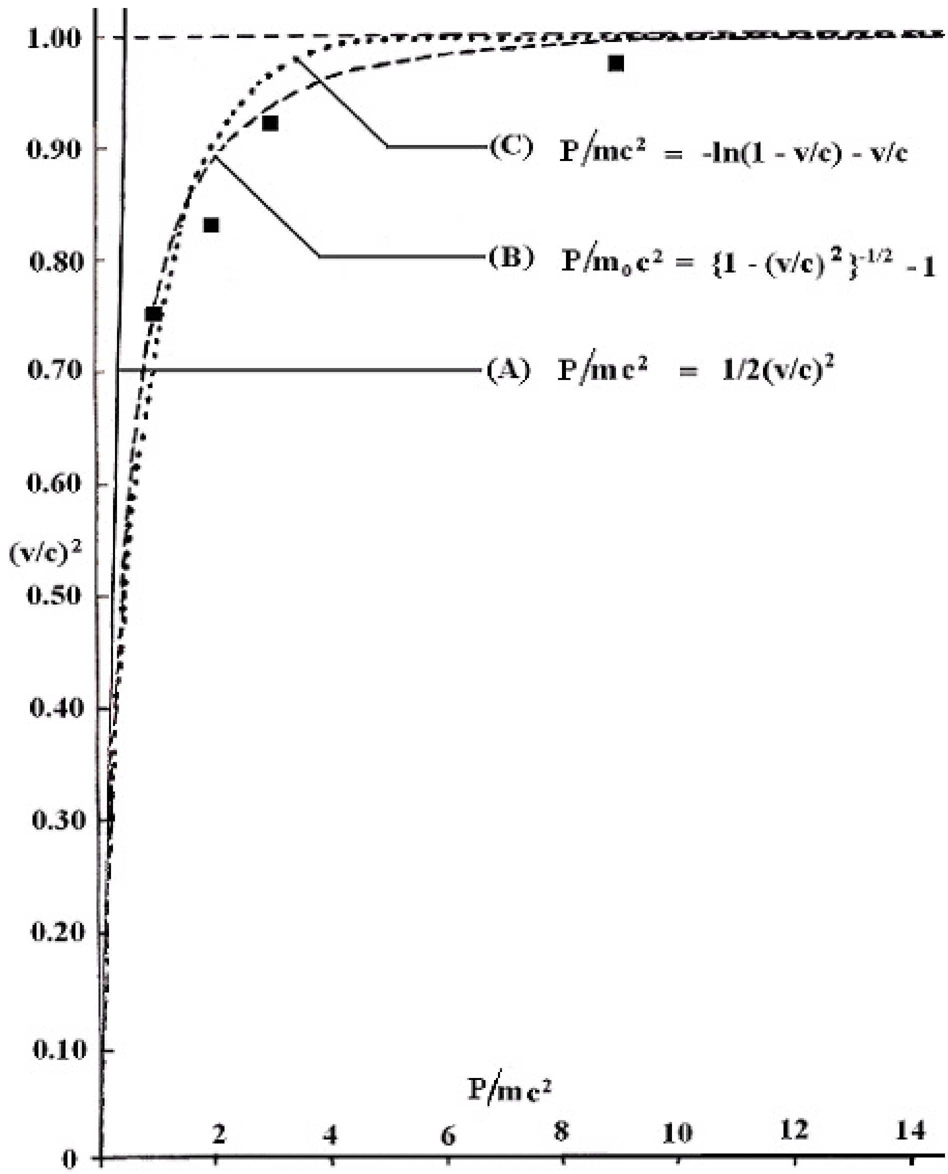

Figure 8.3. v^2/c^2 (speed squared in units of c^2) against P/mc^2 (potential energy in units of mc^2) for an electron of mass *m* accelerated from zero initial speed (*c* is speed of light); the solid line (A) according to classical electrodynamics equation (8.6), the dashed curve according to relativistic electrodynamics (equation 8.11) and the dotted curve (C) according to *radiational electrodynamics* (equation 8.23). The solid squares are the result of Bertozzi's experiment (Table 8.2).

8.7 Radius of revolution in a circle

In classical electrodynamics, the radius of circular revolution of an electron of charge *–e* and mass $m = m_o$ in an electrostatic field of magnitude E due to a central source charge, as given by equation (8.9), is:

$$r_o = \frac{m_o v^2}{eE} \tag{8.31}$$

where r_o is the classical radius. In relativistic electrodynamics, the radius of circular revolution, as given by equation (8.13), is:

$$r = \frac{mv^2}{eE} = \frac{m_o v^2}{eE\sqrt{1-\frac{v^2}{c^2}}} = \gamma r_o \tag{8.32}$$

Radiational electrodynamics (equation 8.30) gives the radius of circular revolution as:

$$r = \frac{mv^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_o v^2}{eE\sqrt{1-\frac{v^2}{c^2}}} = \gamma r_o \tag{8.33}$$

Thus relativistic electrodynamics (equation 8.32) gives the same expression, for the radius of circular revolution of an electron, as radiational electrodynamics (equation 8.33)

8.8 Conclusion

Roger's experimental results (Table 8.1) are in agreement with relativistic electrodynamics (equation 8.11) and *radiational electrodynamics* (equation 8.23) for an electron of charge *–e* and mass *m* revolving with speed *v* in a circle of radius *r*, under the influence of a radial electrostatic field of magnitude *E*. It could, therefore, be concluded that the predicted increase of mass of a particle with its speed, might have been confirmed.

The relativistic mass *"m"* in equation 1, is not a physical mass, but the ratio of the force *(eE)* on a stationary charged particle to the centripetal acceleration (v^2/r) in circular motion. This ratio may be infinite, for motion in a circle of infinite radius (a straight line). Thus, we have the speed of light as the limit, without infinite mass.

Relativistic electrodynamics and *radiational electrodynamics* merge to classical at very low speeds. Relativistic electrodynamics and *radiational electrodynamics* give zero acceleration at the speed of light. In relativistic electrodynamics, an electron cannot attain the speed of light *c*, no matter the magnitude of accelerating potential. In *radiational electrodynamics* an electron is easily accelerated to the speed of light by a potential energy of 15 Mev or over.

Relativistic electrodynamics (equation 8.32) gives the same expression, for radius of circular revolution of an electron, as *radiational electrodynamics* (equation 8.33). While the radius can increase and become infinite for motion in a straight line, the mass (equation 8.1), a physical quantity, cannot expand and become infinitely large while its dimension reduces to zero.

Decrease of accelerating force with speed, as predicted by *radiational electrodynamics*, gives the same effect as apparent increase of mass with speed, as far as circular revolution of an electron is concerned. What actually increases with speed is the radius of revolution in circular motion as expressed in equation 8.33. This radius can become infinite with speed, for rectilinear motion. So, if *radiational electrodynamics* is valid, the relativistic mass-velocity formula is applicable only to circular revolution of an electron round a centre of force of attraction. Applying this formula, to rectilinear motion of electrons in an accelerator, is wrong.

Bertozzi's experimental results (Table 8.2) are in agreement with relativistic electrodynamics (equation 8.11) and *radiational electrodynamics* (equation 8.23) for an electron in rectilinear motion, as depicted in Fig. 8.3. The two systems of electrodynamics demonstrate the speed of light *c* as a limit; relativistic electrodynamics on the basis of mass, of a moving particle, becoming infinitely large at the speed of light and *radiational electrodynamics* on the basis of accelerating force, on a charged particle, reducing to zero at that speed.

The question now is: "Which one of the electrodynamics is correct?" The answer may be found in the motion of electrons decelerated from the speed of light *c*. According to relativistic electrodynamics, an electron, moving at the speed of light (with infinite kinetic energy), cannot be stopped by any finite force. In *radiational electrodynamics*, an electron moving at the speed of light, is easily brought to rest (equation 8.28), on entering a decelerating field, after gaining potential energy equal to $0.307mc^2$ and radiating energy equal to $0.193mc^2$. The electron may then be accelerated to reach a speed equal to $-c$. An electron moving at the speed of light, being stopped at all, contradicts the special relativity.

8.9 References

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