6. ON THE ENERGY AND MASS OF ELECTRIC CHARGES IN A BODY

Abstract

It is shown that the mass M of a distribution of N positive and negative electric charges $(\pm Q_i)$, is given by the sum:

$$M = \mu_o \varepsilon_o \sum_{i=1}^{N} Q_i V_i = 2\mu_o \varepsilon_o W = \frac{2W}{c^2}$$

where μ_o is the permeability, ε_o the permittivity and c the speed of light in a vacuum, V_i is the electrostatic potential at the position of Q_i the ith charge and W the electrostatic energy. The total energy E of mass M moving at speed v, is:

$$E = W + \frac{1}{2}Mv^2 = \frac{1}{2}M\left(c^2 + v^2\right)$$

in contrast to the relativity theory which makes $E = Mc^2$. The derivation of mass in terms of electric charges in a body comes from an explanation of the origin of inertia.

Keywords: Electric charge, energy, mass, relativity.

6.1 Introduction

Einstein's [1, 2] most famous formula of special relativity, the mass-energy equivalence law, gives the total energy content E of a particle of mass m and rest mass m_o moving with speed v, as:

$$E = mc^2 = \frac{m_o c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{6.1}$$

In this equation, which also expresses the mass-speed formula, the mass m of the particle and total energy content E becoming infinitely large at the speed of light c is a difficulty. In spite of this problem, equation (6.1) is the most celebrated formula in the world.

Equation (6.1) is used by the proponents of special relativity, to explain why an electron, the lightest particle known in nature, cannot be accelerated beyond the speed of light c. In fact, electrons are easily accelerated and have been accelerated to the speed of light, through a potential energy of 15 MeV or higher, as demonstrated by Bertozzi using a linear accelerator [3]. Cyclic electron accelerators (betatrons and electron synchrotrons) of over 100 BeV [4] have been built and operated, with electron speed equal to that of light c for all practical purposes. Such "massive" electrons would have increased the weight of the accelerator and, on impact, should have crushed through the target to cause a catastrophe. It is most likely that something else is responsible for restraining the electron, accelerated by an electrostatic field, from going beyond the speed of light c.

The author [5] showed that the mass m of a moving electron remains constant at the rest mass m_o and that it is the accelerating force exerted by an electrostatic field, on a moving charged particle, which decreases with the speed of the particle, becoming zero at the speed of light c. In this respect we have the ultimate speed without infinite mass.

The author [5] also showed that, for an electron of mass m and charge $-e$, revolving with constant speed v in a circle of radius r, under a central electrostatic field of magnitude E, the quantity "m" in equation (6.1) is the ratio $(eE/v^2)r$ of the magnitude of the force (eE) on a stationary electron, to the centripetal acceleration (v^2/r). The centripetal acceleration reduces to zero at the speed of light c. The radius r and the quantity $(eE/v^2)r$ may then become infinite, for motion in a circle of infinite radius (a straight line is an arc of a circle of infinite radius), without any problem of infinitely large masses at the speed of light.

The purpose of this paper is to derive expressions for the electrostatic energy and physical mass of a distribution of equal number of positive and negative electric charges constituting a neutral body of mass M. The total energy of the body of mass M moving with speed v, relative to a stationary observer, is then deduced as the electrostatic energy of the mass plus its kinetic energy. The total energy content is then compared with Einstein's [1, 2] mass-energy equivalence law of the theory of special relativity.

6.2 Energy content of an electric charge distribution

If an isolated electric charge Q assumed any shape or configuration, it is most likely to be a spherical shell of radius a, with all the charges on the surface at the same potential, as in Figure 6.1(a). Such a figure has "force

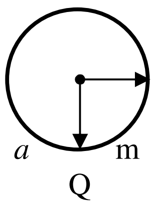

Figure 6.1(a). Uniformly charged spherical shell of fixed radius *a*, total charge *Q* and mass *m*

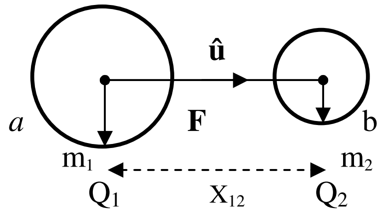

Figure 6.1(b). Two charges Q_1 and Q_2 distance X_{12} apart with force of repulsion ***F*** in the ***û*** direction

of explosion" as well as self energy or intrinsic energy due to the charge being situated in its own potential. The intrinsic energy of the charge *Q* in Figure 6.1(a) is the work *w* done by an external force in assembling the magnitude of the charge from *0* to *Q* as a spherical shell of fixed radius *a*. The intrinsic energy is always positive.

For points outside a uniformly charged spherical surface, it is as if the whole charge is concentrated at the centre. The potential inside and on the spherical surface of radius *a* carrying a charge *q*, is $q/4\pi\varepsilon_o a$ and the work done in increasing the potential from *0* to *U* and building the charge from *0* to *Q* at a fixed radius *a*, by equal infinitesimal amounts *(dq)*, is the intrinsic energy *w*, given by the definite integral:

$$w = \int_0^Q \frac{q}{4\pi\varepsilon_o a}(dq) = \frac{Q^2}{8\pi\varepsilon_o a} = \frac{QU}{2} \qquad (6.2)$$

Equation (6.2) shows that the intrinsic energy *w* is *½Q times the electrostatic potential U in which the charge Q is located*. The energy content is always positive and proportional to the square of the (positive or negative) charge.

Figure 6.1(b) shows two positive electric charges Q_1 and Q_2 of fixed internal radii a_1 and a_2 respectively, separated by a distance X_{12} in a body. The electrostatic energy w_2 of the charges is the sum of the intrinsic energy of each charge being in its own potential and the extrinsic energy due to one charge being in the electrostatic potential of the other, which is expressed in the equation:

$$w_2 = \frac{Q_1^2}{8\pi\varepsilon_o a_1} + \frac{Q_2^2}{8\pi\varepsilon_o a_2} + \frac{Q_1 Q_2}{4\pi\varepsilon_o X_{12}}$$

$$w_2 = \frac{Q_1}{2}\left(\frac{Q_1}{4\pi\varepsilon_o a_1} + \frac{Q_2}{4\pi\varepsilon_0 X_{12}}\right) + \frac{Q_2}{2}\left(\frac{Q_2}{4\pi\varepsilon_o a_2} + \frac{Q_1}{4\pi\varepsilon_o X_{12}}\right) \quad (6.3)$$

For 3 charges, Q_1, Q_2 and Q_3, of respective radii a_1, a_2, and a_3, separated by distances X_{12}, X_{13}, and X_{23} in a body, the total of the intrinsic and extrinsic energies is:

$$w_3 = \frac{Q_1^2}{8\pi\varepsilon_o a_1} + \frac{Q_2^2}{8\pi\varepsilon_o a_2} + \frac{Q_3}{8\pi\varepsilon_o a_3} + \frac{Q_1 Q_2}{4\pi\varepsilon_o X_{12}} + \frac{Q_1 Q_3}{4\pi\varepsilon_o X_{13}} + \frac{Q_2 Q_3}{4\pi\varepsilon_o X_{23}}$$

$$w_3 = \frac{Q_1}{2}\left(\frac{Q_1}{4\pi\varepsilon_o a_1} + \frac{Q_2}{4\pi\varepsilon_0 X_{12}} + \frac{Q_3}{4\pi\varepsilon_o X_{13}}\right) +$$
$$\frac{Q_2}{2}\left(\frac{Q_2}{4\pi\varepsilon_o a_2} + \frac{Q_1}{4\pi\varepsilon_o X_{12}} + \frac{Q_3}{4\pi\varepsilon_o X_{23}}\right) +$$
$$\frac{Q_3}{2}\left(\frac{Q_3}{4\pi\varepsilon_o a_3} + \frac{Q_1}{4\pi\varepsilon_o X_{13}} + \frac{Q_2}{4\pi\varepsilon_o X_{23}}\right)$$

Where the charges are positive and negative, the extrinsic energies are positive and negative and may cancel out so that:

$$w_3 = \frac{1}{2}\left\{Q_1\left(U_1 + \Lambda_1\right) + Q_2\left(U_2 + \Lambda_2\right) + Q_3\left(U_3 + \Lambda_3\right)\right\}$$

$$w_3 = \frac{1}{2}\left(Q_1 V_1 + Q_2 V_2 + Q_3 V_3\right)$$

For a number *N* of charges in a body, the total electrostatic energy is the sum *W* given by:

$$W = \frac{1}{2}\sum_{i=1}^{N} Q_i\left(U_i + \Lambda_i\right) = \frac{1}{2}\sum_{i=1}^{N} Q_i V_i \quad (6.4)$$

where U_i is the intrinsic potential due the ith charge at the point of location of the ith charge, Λ_i is the total extrinsic potential due to all the other charges (excluding the ith charge) at the position of the ith charge and V_i is the electrostatic potential due to all the charges (including the ith charge) at the point of location of the ith charge. It should be noted that the product $(Q_i V_i)$ at any point, outside a charge $(Q_i = 0)$, is zero.

In a neutral body, containing equal number of positive and negative electric charges, the products, $Q_i U_i$, are all positive and add up. The potentials Λ_i are positive or negative and the sum of the products $Q_i\Lambda_i$ may be zero, so that equation (4.4) becomes:

$$W = \frac{1}{2}\sum_{i=1}^{N} Q_i\left(U_i + \Lambda_i\right) = \frac{1}{2}\sum_{i=1}^{N} Q_i U_i \qquad (6.5)$$

Equations (6.4) and (6.5) will be used to derive an expression for the energy content of a body of mass M.

6.3 Mass of an isolated electric charge and charge distribution

An isolated positive electric charge of magnitude Q moving in a straight line with velocity $\boldsymbol{v}$ and acceleration $(d\boldsymbol{v}/dt)$ at time t, is associated with a circular magnetic field of intensity $\boldsymbol{H}$ and an electrodynamic field of intensity $\boldsymbol{E}_a$, as shown in Figure 6.2.

The magnetic flux intensity, $\boldsymbol{B} = \mu_o\boldsymbol{H}$, due to an electric charge of magnitude Q, with its electrostatic field of intensity $\boldsymbol{E}$, (Figure 6.2) moving at velocity $\boldsymbol{v}$ in free space or vacuum, is given as a vector (cross) product, by Biot and Savart law of electromagnetism [6], as a vector equation, thus:

$$\mathbf{B} = \mu_o\varepsilon_o\mathbf{v}\times\mathbf{E} = -\mu_o\varepsilon_o\mathbf{v}\times\nabla\phi \qquad (6.6)$$

where μ_o is the permeability and ε_o the permittivity of free space or vacuum, ϕ (a scalar) is the instantaneous electric potential at any point due to the charge and $\boldsymbol{E} = -\nabla\phi$, is the electrostatic field intensity, as given by Coulomb's law of electrostatics:

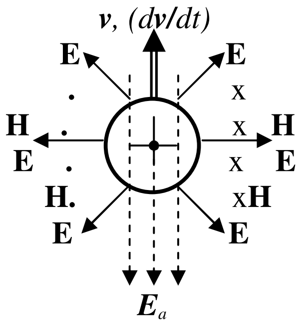

Figure 6.2 An isolated electric charge Q and its electrostatic field $\boldsymbol{E}$ moving in a straight line with velocity $\boldsymbol{v}$ and acceleration $(d\boldsymbol{v}/dt)$ at time t, generating magnetic field of intensity $\boldsymbol{H}$ and electrodynamic field of intensity $\boldsymbol{E}_a$.

In Figure 6.2, the magnetic field ***H*** is out of the page on the left and into the page on the right. The electrodynamic field $\boldsymbol{E}_a$ points downwards, opposite to the direction of acceleration. A charge in acceleration is always associated with an opposing electrodynamic field.

Vector transformation of equation (6.6) gives:

$$\mathbf{B} = -\mu_o \varepsilon_o \mathbf{v} \times \nabla \phi = \mu_o \varepsilon_o \nabla \times (\phi \mathbf{v})$$

Here ∇ denotes the "gradient" of a scalar quantity, $\nabla\times$ depicts the "curl" of a vector quantity and the curl of velocity $\boldsymbol{v}$, $(\nabla\times \boldsymbol{v}) = 0$. Faraday's law of electromagnetic induction [7] gives:

$$\nabla \times \mathbf{E}_a = -\frac{d\mathbf{B}}{dt} = -\mu_o \varepsilon_o \nabla \times \left(\phi \frac{d\mathbf{v}}{dt} \right)$$

$$\mathbf{E}_a = -\mu_o \varepsilon_o \phi \frac{d\mathbf{v}}{dt} \tag{6.7}$$

The idea put forward here is that the electrodynamic field $\boldsymbol{E}_a$ (equation 6.7) acts internally on the self-same charge Q to produce the reactive force, inertial force or reverse effective force, equal and opposite to the accelerating force. For an isolated uniform spherical shell of electric charge Q and mass m, equation (6.7) and Newton's Second Law of Motion give the reverse effective force as:

$$\mathbf{E}_a Q = -\mu_o \varepsilon_o Q U \frac{d\mathbf{v}}{dt} = -m \frac{d\mathbf{v}}{dt}$$

$$m = \mu_o \varepsilon_o Q U \tag{6.8}$$

where U is the electrostatic (intrinsic) potential due the charge Q at the point of location of the same charge. The derivation of mass m in equation (6.8) gives an explanation of inertia as experienced by a body under acceleration or deceleration.

For a uniform spherical shell of charge Q, radius a and mass m, equations (6.2) for intrinsic energy and equation (6.8), give:

$$w = \frac{Q^2}{8\pi\varepsilon_o a} = \frac{QU}{2} = \frac{m}{2\mu_o \varepsilon_o} = \frac{1}{2} mc^2 \tag{6.9}$$

$$m = \frac{\mu_o Q^2}{4\pi a} \tag{6.10}$$

where μ_o is the permeability, ε_o the permittivity and $c = (\mu_o \varepsilon_o)^{-1/2}$ is the speed of light in a vacuum, as discovered in 1873 by Maxwell [8].

Equations (6.8) and (6.10), which give the mass of an isolated electric charge Q, should be noted as it has implications in the gravitational force of attraction between bodies or masses composed of equal number of positive and negative electric charges in space.

For a rigid neutral body containing *N/2* positive electric charges and N/2 negative electric charges moving with the same acceleration, *(dv/dt)*, the total electrodynamic field generated at an external point, comes to zero (0). This is obvious as the constituent electric charges generate equal and opposite fields.

For a distribution of N positive and negative electric charges, constituting a body, equation (6.8) gives the mass M as the sum:

$$M = \mu_o \varepsilon_o \sum_{i=1}^{N} Q_i \left(U_i + \Lambda_i \right) \tag{6.11}$$

where U_i is the intrinsic potential due the ith charge at the point of location of Q_i the ith charge and Λ_i is the extrinsic potential due to all the other charges (excluding the ith charge) at the position of the ith charge. The products Q_iU_i are all positive but the products $Q_i\Lambda_i$ are positive or negative and their sum may be zero.

For a body made up of N electric charges, equations (6.4) for W and equation (6.11) for the mass M, give:

$$W = \frac{1}{2} \sum_{i=1}^{N} Q_i \left(U_i + \Lambda_i \right) = \frac{M}{2\mu_o \varepsilon_o} = \frac{1}{2} Mc^2 \tag{6.12}$$

According to equation (6.12), the work done W in creating a distribution of charges constituting a body of mass M, is $W = ½Mc^2$, equal to the electrostatic energy of the mass. Where mass is independent of speed, the kinetic energy of a body of mass M, moving with speed v, in accordance with classical (Newtonian) mechanics, is $K = ½Mv^2$. The total energy content E, is:

$$E = W + K = \frac{M}{2} \left(c^2 + v^2 \right) \tag{6.13}$$

Equation (6.13) is in contrast to Einstein's mass-energy equivalence law of special relativity [1, 2] as expressed in equation (6.1). Equation (6.13) is what this paper set out to derive.

6.4 Conclusion

The derivation of equation (6.8) gives clarification for the origin of inertia as electrical and internal to an accelerated or decelerated body. This is a new and important discovery in physics, particularly electrodynamics.

Equation (6.10), expressing the mass of a charged particle, in the form of a spherical shell of radius a, is instructive. If the electric charge remains constant with speed of the particle, it is reasonable to conclude that the mass should similarly remain constant, contrary to special relativity as expressed in equation (6.1).

Equations (6.1) and (6.13), for a stationary particle, differ by a factor of *2*. However, each equation gives a body of rest mass M_o as the source of a tremendous amount of energy locked up in the particles. As to which equation is correct, remains to be decided. If the decision is in favour of equation (6.13), then it would have a tremendous impact in redirecting the course of modern physics.

In equation (6.1) the kinetic energy is contained in the increase of mass of the particle, which becomes infinitely large at the speed of light c. In equation (6.13), mass M remains constant at the rest mass M_o and the kinetic energy reaches a maximum value equal to $½M_oc^2$ at the speed of light c. Thus, bodies may be accelerated to the speed of light, without mass becoming infinitely large.

The mass-energy equivalence formula, as given by equations (6.9) and (6.13), is more realistic than equation (6.1) for a particle, such as an electron, that can easily be accelerated to the speed of light c. According to equation (6.13), the maximum energy content of an electron of mass m, moving with the speed of light c, is $E_m = mc^2$. Such an electron is easily brought to rest, losing kinetic energy equal to $½mc^2$. The electron can impinge on a target and may recoil without causing any damage. If the mass were infinite, its impact on a target would be destructive.

We conclude that mass (equation 6.10) is not a fundamental quantity. The four fundamental quantities are better put as *Length (L)*, *Time (T)*, *Electric Charge (Q)* and *Electric Potential (V)*. This system, the *(Metre-Second-Coulomb-Volt)* system, gives the dimension of *mass (M)* as $[L^{-2}T^{2}QV]$, that of *Permittivity (ε)* is $[L^{-1}QV^{-1}]$, *Permeability (μ)* is $[L^{-1}T^{2}Q^{-1}V]$, M*agnetic Field (H)* is $[L^{-1}T^{-1}Q]$, *Magnetic Flux (Ψ)* is $[TV]$, *Resistance (R)* is $[TQ^{-1}V]$, *Resistivity (ρ)* is $[LTQ^{-1}V]$, *Conductivity (σ)* is $[L^{-1}T^{-1}QV^{-1}]$, *Capacitance (C)* is $[QV^{-1}]$, *Inductance (L)* is $[T^{2}Q^{-1}V]$, *Force (F) is [L^{-1}QV]*, *Energy (E)* is *[QV]*, *Momentum (p)* is *[L^{-1}TQV]* and *Angular Momentum (L)* is *[TQV]*. There is no fractional exponent of a fundamental quantity in the dimension of any quantity.

6.5 References

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