5. ON THE SPEED OF LIGHT IN A MOVING MEDIUM

Abstract

The speed w of transmitted light, at normal incidence, in a medium of refractive index μ_2 moving with speed v in another medium of refractive index μ_1, is derived as:

$$w = \frac{c}{\mu_2} + v\left(1 - \frac{\mu_1}{\mu_2}\right)$$

where c is the speed of light in a vacuum. The equation for w is applied to give a non-relativistic explanation of the result of Fizeau's experiment without recourse to special relativity.

Keywords: Fizeau's experiment, light, medium, speed.

5.1 Introduction

Euclid (330 – 280 B.C.), who discovered the law of formation of images by mirrors, probably knew the law of reflection of light [1]. Snell discovered the law of refraction of light in about 1620 [1]. When a ray of light, propagated in a *medium 1*, reaches the boundary with another *medium 2*, reflection and refraction occur. Fig. 5.1 depicts a ray ***SP***, emitted with velocity ***s*** from a stationary source *S*, incident at a point *P* on the boundary of two media.

A reflected ray ***PR*** is one propagated with velocity ***u*** from the boundary in the same medium as the incident ray ***SP***. A refracted ray ***PT*** is one transmitted and propagated with velocity ***w*** in the second medium. The angle of incidence is ι, where $(\pi - \iota)$ is the angle between the directions of propagation of the incident ray ***SP*** and the normal ***OPN*** to the boundary at the point of incidence *P*. The angle of reflection ρ is the angle between the directions of propagation of the reflected ray ***PR*** and the normal. The angle of refraction is τ, where $(\pi - \tau)$ is the angle between the directions of propagation of the refracted ray ***PT*** and the normal at *P*.

5.1.1 Laws of reflection of light

Yavorsky and Detlaf [2] recapitulated the laws of reflection and refraction of light, for a stationary medium. The laws of reflection, with reference to Figure 5.1, are:

(i) The incident ray ***SP***, the reflected ray ***PR*** and the normal ***OPN***, at the point of incidence *P*, are coplanar.

(ii) The angle of reflection ρ is equal to the angle of incidence ι.

5.1.2 Laws of refraction of light

The laws of refraction of light, with $v = 0$ in Figure 5.1, are:

(i) The incident ray ***SP***, the refracted ray ***PT*** and the normal ***OPN***, at the point of incidence *P*, are coplanar.

(ii) The ratio of the sine of angle of refraction to the sine of angle of incidence, for light of a given wavelength, is equal to the ratio of the speeds of light *w* and *s* in the media.

The second law of refraction is called Snell's law, which can be deduced from Fermat's principle [3].

The relative indices of refraction μ_1 and μ_2 of *media 1* and *2* respectively, are defined, for a stationary medium, as the ratio:

$$\frac{w}{s} = \frac{\sin \tau}{\sin \iota} = \frac{\mu_1}{\mu_2} \tag{5.1}$$

In a vacuum, $\mu = 1$ and $\mu_1 s = \mu_2 w = c$, the speed of light. If medium 1 is a vacuum, $\mu_2 = \mu$ is the absolute index of refraction.

Where $\mu_1 > \mu_2$ (Figure 5.1), that is transmission from a denser medium to a less dense medium, such as glass to air or vacuum, the angle of refraction τ is greater than the angle of incidence ι,. Total internal reflection occurs if $\tau > \pi/2$ radians. The value υ of ι at which $\tau = \pi/2$ radians is called the *critical angle*, such that $\sin\upsilon = \mu_2/\mu_1 < 1$.

5.1.3 Dispersion of light

The index of refraction depends on the wavelength of light radiation under consideration. The shortened form of Cauchy's empirical formula [4] gives an approximate relationship between the index of refraction μ of a transparent medium and the wavelength λ of light, as:

$$\mu = a + \frac{b}{\lambda^2} \tag{5.2}$$

where *a* and *b* are constants for the medium. The decrease of μ with λ causes the dispersion of white light into the colours of the rainbow.

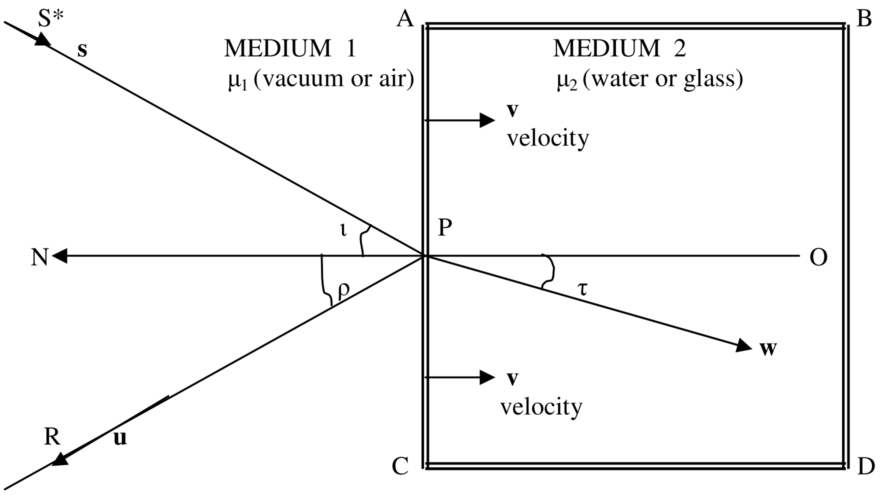

Figure 5.1 Reflection and refraction of a ray of light emitted from a source *S* with velocity ***s*** in medium *1*, incident at a point *P* on the surface of medium *2* moving with velocity ***v***. The ray ***PR*** is reflected with velocity ***u*** and the refracted ray ***PT*** transmitted in medium 2 with velocity ***w***. The velocity vector ***v*** is in the opposite direction of the normal ***OPN*** at *P*.

5.2. Speed of light reflected from a moving medium

Figure 5.1, drawn in accordance with the first law of reflection and refraction, shows a monochromatic ray of light in *medium 1* propagated with velocity ***s*** from a stationary source *S*. The ray is incident at a point *P* on the plane surface of denser *medium 2 (ABCD)*, such as glass or water, moving with velocity ***v*** (relative to a stationary observer) in a direction opposite to the normal ***OPN*** at *P*. The reflected ray is propagated with velocity ***u*** in the same *medium 1* as the incident ray. The angle of incidence is ι and the angle of reflection is ρ. The *medium 2* may be considered as stationary by giving each of the rays a velocity equal to –***v***. The ratio of the magnitudes of the relative velocities of the reflected ray and the incident ray, with respect to the medium, is put as:

$$\frac{|\mathbf{u}-\mathbf{v}|}{|\mathbf{s}-\mathbf{v}|}=\frac{\mu_1}{\mu_1}=1 \qquad (5.2)$$

With reference to Figure 5.1, the ratio is obtained as:

$$\frac{\sqrt{u^2+v^2+2uv\cos\rho}}{\sqrt{s^2+v^2-2sv\cos\iota}} = \frac{u^2+v^2+2uv\cos\rho}{s^2+v^2-2sv\cos\iota} = 1 \tag{5.3}$$

If $v = 0$, obviously the incident and reflected rays, being in the same medium, will have the same speed $u = s$.

At normal incidence and the *medium 2* moving with speed v, $\iota = \rho = 0$, we obtain the equation:

$$\frac{u+v}{s-v} = 1$$

$$u = s - 2v \tag{5.4}$$

Where the first medium is a vacuum, s is the speed of light c. Equation (5.4) is exactly as obtained if the ray of light is considered as a "stream of particles" propagated with speed c. The "particles" or photons would impinge normally on the moving medium at the point of incidence P, on perfectly elastic collisions, and recoil with speed u given by:

$$u = c - 2v \tag{5.5}$$

This is the idea behind "ballistic propagation of light", in accordance with Newton's law of restitution [5].

5.3. Reflection angle from a moving medium

Equation (5.3), with $s = c$, gives the cosine of the angle of reflection, for a medium moving along the normal with speed v in a vacuum, as:

$$\cos\rho = \frac{1}{2uv}\left(c^2 - u^2 - 2cv\cos\iota\right) \tag{5.6}$$

Since $\rho = 0$ where $\iota = 0$ and $\rho = \pi/2$ radians where $\iota = \pi/2$ radians, it can be inferred that the law of reflection ($\rho = \iota$) applies irrespective of the speed of the reflecting medium moving along the normal with speed v. At the point of incidence equation (5.6) then becomes:

$$\cos\rho = \cos\iota = \frac{c-u}{2v} \tag{5.7}$$

5.4 Speed of light transmitted in a moving medium

A ray of light emitted with velocity ***s*** is incident on the surface of a transparent medium moving with velocity ***v*** along the normal as in Fig. 5.1. The ray is transmitted through the medium with velocity ***w*** at the angle of refraction τ. The refractive indices are defined in terms of the ratio of magnitudes of relative velocities of the refracted ray and the incident ray. The ratio of magnitudes of the relative velocity ***(w – v)*** and relative velocity ***(s – v)*** is obtained, thus:

$$\frac{|\mathbf{w}-\mathbf{v}|}{|\mathbf{s}-\mathbf{v}|}=\frac{\mu_1}{\mu_2}$$

With reference to Figure 5.1, the ratio is obtained as:

$$\frac{\sqrt{w^2+v^2-2wv\cos\tau}}{\sqrt{s^2+v^2-2sv\cos\iota}}=\frac{\mu_1}{\mu_2} \qquad (5xx)$$

At normal incidence, $\iota = \tau = 0$ and we obtain:

$$\frac{w-v}{s-v}=\frac{\mu_1}{\mu_2}$$

$$w\mu_2 - v\mu_2 = s\mu_1 - v\mu_1$$

$$w=\frac{s\mu_1}{\mu_2}+v\left(1-\frac{\mu_1}{\mu_2}\right) \qquad (5.8)$$

$$w=\frac{c}{\mu_2}+v\left(1-\frac{\mu_1}{\mu_2}\right) \qquad (5.9)$$

where $s\mu_1 = c$ is the speed of light in a vacuum. For a medium moving in a vacuum where the refractive index $\mu_1 = 1$, we obtain the speed w of transmission of light in the medium, as:

$$w = \frac{c}{\mu} + v\left(1 - \frac{1}{\mu}\right) \tag{5.10}$$

Equation (5.10) is a significant result of this paper. It is used to give a non-relativistic explanation of the result of Fizeau's experiment, which measured the speed of light in moving water, without recourse to the theory of special relativity.

5.5. Fizeau's experiment

A schematic diagram of the apparatus of Fizeau's experiment [6] is shown in Figure 5.2. Carried out in the 1850s, it was one of the most remarkable experiments in physics. In this experiment, light from a source was sent in two opposite directions through transmission and reflection by four half-silvered mirrors M_1 – M_4. One beam travelled downstream (from M_1) through moving water and the other travelled upstream (from M_4) in the same water. By an ingenious arrangement of the mirrors the two beams were made to recombine and be observed in an interferometer. An interference pattern, as observed in an interferometer, resulted from the difference in the time taken for the two beams to travel the same path, partly in moving water.

Various explanations have been given for the result of Fizeau's experiment. One accepted explanation was that the velocity of light in the moving medium (water) was increased or decreased in accordance with the *relativistic rule for addition of velocities* based on constancy of the speed of light relative to a moving observer or a moving object. A new explanation is proposed in this paper, on the basis of equation (5.10), outside Einstein's theory of special relativity [7, 8].

5.6 Non-relativistic explanation of the result of Fizeau's experiment

According to the Galilean-Newtonian relativity of classical mechanics, the speed of light in a medium moving in a vacuum with velocity $\boldsymbol{v}$, in the opposite direction of the normal, is given in terms of the refractive index, by equation (5.10).

Equation (5.10) gives the speed w of transmission of light (downstream). If the medium is water of index of refraction μ moving with velocity $\boldsymbol{v}$ in a vacuum (Figure 5.2), the time taken for the beam that

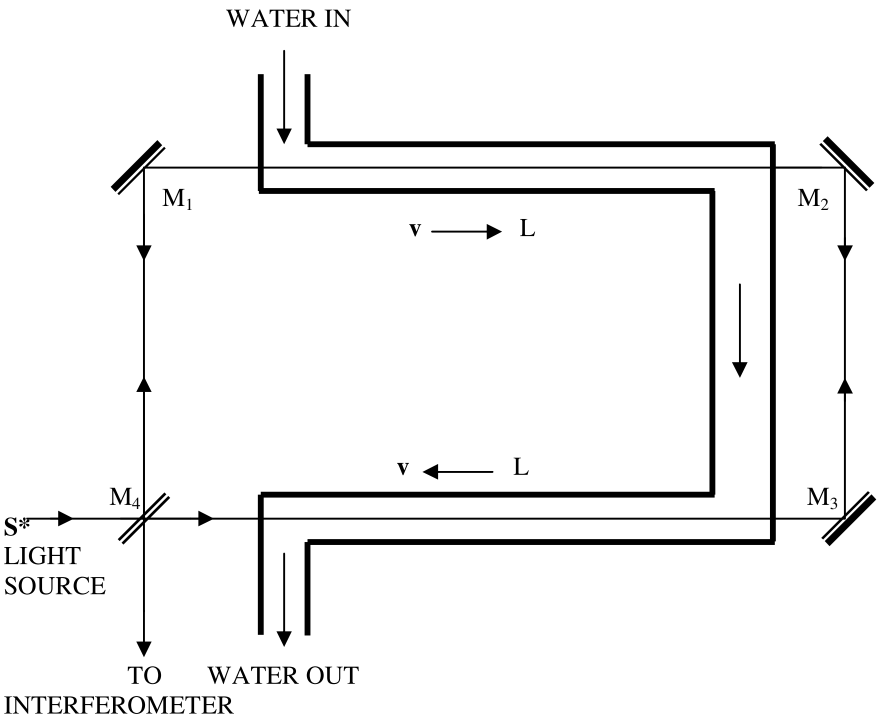

Figure 5.2. Schematic diagram of the apparatus of Fizeau's experiment

is going downstream to cover the distance *2L* (where the magnitude *v* is very small compared with the speed of light *c* and, therefore, v^2/c^2 can be neglected), is obtained as the equation:

$$t_1 = \frac{2L}{\dfrac{c}{\mu} + v\left(1 - \dfrac{1}{\mu}\right)} \approx \frac{2\mu L}{c}\left\{1 - \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\} \qquad (5.11)$$

For the beam going upstream, with velocity **–*v*** (with respect to a stationary observer), the longer transit time is:

$$t_2 = \frac{2L}{\dfrac{c}{\mu} - v\left(1 - \dfrac{1}{\mu}\right)} \approx \frac{2\mu L}{c}\left\{1 + \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\} \qquad (5.12)$$

The time difference $(\Delta t) = (t_2 - t_1)$ between the two beams, traversing the same path of length *2L* (downstream or upstream) in moving water, is:

$$t_2 - t_1 = \frac{2\mu L}{c}\left\{1 + \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\} - \frac{2\mu L}{c}\left\{1 - \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\}$$

$$\Delta t = \frac{4Lv\mu^2}{c^2}\left(1 - \frac{1}{\mu}\right) \tag{5.13}$$

The fringe shift δ_x, for light of wavelength λ, is obtained as:

$$\delta_x = \frac{c\Delta t}{\lambda} = \frac{4Lv\mu^2}{\lambda c}\left(1 - \frac{1}{\mu}\right) \tag{5.14}$$

In the experiments performed by Fizeau [6], $L = 3\ m$, $v = 7\ m/sec.$, $\lambda = 6 \times 10^{-7}\ m$ (yellow light), $c = 3 \times 10^{8}\ m/sec$ and $\mu = 4/3$ (for water). The fringe shift δ_x is obtained as ≈ 0.2, which was easily observable and measurable in the interferometer.

Michelson and Morley in 1886 and later P. Zeeman and associates in 1915 repeated Fizeau's experiment with greater precision in which the interferometer could measure a fringe shift as low as *0.01*. The influence of the motion of the medium on the propagation of light has thus been verified. As to which is the correct explanation, the theory of special relativity, according to Einstein or equation (5.14), in accordance with Galilean relativity of classical mechanics, remains to be seen.

5.7. Relativistic explanation of the result of Fizeau's experiment

According to Galileo's *velocity addition rule*, if you move with velocity ***u*** relative to a medium moving with velocity ***v*** (passenger jogging with velocity **u** in a ship moving with velocity **v**) your velocity relative to a stationary observer is vector sum ***s***, given by:

$$\mathbf{s} = \mathbf{u} + \mathbf{v} \tag{5.15}$$

Here the velocities ***u*** and ***v*** are vectors that can be of any magnitude and in any direction. If the velocities are in the same direction, the speed relative to a stationary observer is the magnitude $s = u + v$. This is the Galilean principle of relativity, which is in agreement with observation and natural sense.

According to Einstein's *velocity addition rule,* if you move with velocity $\boldsymbol{u}$ relative to a medium moving with velocity $\boldsymbol{v}$ (as a jogger running with velocity $\mathbf{u}$ in a ship moving with velocity $\mathbf{v}$), the magnitude of your velocity, relative to a stationary observer, is:

$$s = \frac{u+v}{1+\frac{uv}{c^2}} \tag{5.16}$$

How the speed of light c in a vacuum comes into (5.16), is perplexing. In vector form, equation (5.16) may be expressed as:

$$\mathbf{s} = \frac{\mathbf{u}+\mathbf{v}}{1+\frac{\mathbf{u}.\mathbf{v}}{c^2}} \tag{5.17}$$

where $\boldsymbol{u.v}$ is a scalar product.

In equation (5.17), $\boldsymbol{u}$ and $\boldsymbol{v}$ must be collinear to give equation (5.16). In reality, the jogger should be able to run with velocity $\boldsymbol{u}$ in any direction relative to $\boldsymbol{v}$. If $u = c$ (speed of light) or $u = v = c$, the speed s remains as c. Equation (5.16), more than anything else, had lent support to the principle of constancy of the speed of light, in accordance with the theory of special relativity. For speeds much less than c, or if c is infinitely large, Einstein's relativistic formula (equation 5.16) reduces to the classical formula (equation 5.15).

According to the theory of special relativity, the velocity of light (jogger), relative to the surface of a medium (ship) moving in a vacuum with velocity $\boldsymbol{v}$, remains as a constant $\boldsymbol{c}$. The velocity of light (jogger) within and with respect to the medium (ship) of refractive index μ is $\boldsymbol{c/\mu}$. Einstein's *velocity addition rule*, with $u = c/\mu$, gives the magnitude of velocity $\boldsymbol{w}$ of light in the moving medium (with respect to a stationary observer) as:

$$w = \frac{\frac{c}{\mu}+v}{1+\frac{cv}{\mu c^2}} = \frac{\frac{c}{\mu}+v}{1+\frac{v}{\mu c}} \approx \frac{c}{\mu}+v\left(1-\frac{1}{\mu^2}\right) \tag{5.18}$$

Equation (5.18), compared with (5.10), is used to obtain the transit time difference between the two beams in Fizeau's experiment, and thereby explain the result from the relativistic point of view.

The transit time of the beam going downstream, with speed v very small compared with c, is:

$$t_1 = \frac{2L}{\frac{c}{\mu} + v\left(1 - \frac{1}{\mu^2}\right)} \approx \frac{2\mu L}{c}\left\{1 - \frac{\mu v}{c}\left(1 - \frac{1}{\mu^2}\right)\right\}$$

$$t_1 \approx \frac{2L\mu}{c}\left(1 + \frac{v}{\mu c} - \frac{\mu v}{c}\right)$$

The transit time of the beam going upstream (with speed $-v$) is:

$$t_2 = \frac{2L}{\frac{c}{\mu} - v\left(1 - \frac{1}{\mu^2}\right)} \approx \frac{2\mu L}{c}\left\{1 + \frac{\mu v}{c}\left(1 - \frac{1}{\mu^2}\right)\right\}$$

$$t_2 \approx \frac{2L\mu}{c}\left(1 - \frac{v}{\mu c} + \frac{\mu v}{c}\right)$$

The time difference between the beam going downstream and the other going upstream is obtained as:

$$\Delta t = t_2 - t_1 \approx \frac{4Lv\mu^2}{c^2}\left(1 - \frac{1}{\mu^2}\right) \tag{5.19}$$

The fringe shift δ_y, for light of wavelength λ, is obtained as:

$$\delta_y = \frac{c\Delta t}{\lambda} = \frac{4Lv\mu^2}{\lambda c}\left(1 - \frac{1}{\mu^2}\right) \tag{5.20}$$

This δ_y is larger than the fringe shift δ_x as given by equation (5.14).

5.8 Conclusion

A ray of light has the effect of a "stream of particles" transmitted at the speed of light c in the direction of propagation. The "stream of particles", which impinge on the surface of a medium exert pressure upon reflection from the surface. Reflection of light can be treated as the recoil of "moving particles", under perfectly elastic conditions, obeying Newton's law of restitution [5].

The treatment of reflection and refraction of light in this paper, clearly demonstrate the relativity of the velocity of light with respect to a moving medium, in accordance with Galileo's relativity of classical mechanics. The result of Fizeau's experiment [6] is not a direct consequence of the relativistic *velocity addition rule* but due to the effect of motion of the transmission medium on the speed of light.

In equations (5.6) and (5.7) it is shown that the law of reflection, that is angle of incidence being equal to the angle of reflection, applies irrespective of the speed of the reflecting medium moving along the normal direction. Equation (5.7) is a simple and new expression subject to experimental verification.

In equations (5.10) and (5.18) the speed v, of the moving medium, can take any value between 0 and $\pm c$. For $v = 0$, both equations give the speed of light in the medium $w = c/\mu$, as expected. For $\mu = 1$ or $v = c$, both equations give $w = c$, also as expected. For $v = -c$, equation (5.10) gives $w = c(2/\mu - 1)$, which is reasonable, but the relativistic equation (5.18) gives $w = c(1/\mu^2 + 1/\mu - 1)$, which may be negative as $1 < \mu < 2$.

Equation (5.20) is another good example of Beckmann's *correspondence theory* [9], whereby the desired result is obtained mathematically but based on the wrong underlying principles. Fizeau's experiment might as well have verified the fringe shift in equation (5.14), rather than the relativistic equation (5.20), for the transmission of light in a moving medium. Curt Renshaw [10] analysed the results of several experiments conducted to measure the effect of the speed of a medium on the speed of transmission of light in the medium and he concluded that the results could be explained without invoking special relativity.

5.9 References

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