3. A NUCLEAR MODEL OF THE HYDROGEN ATOM OUTSIDE QUANTUM MECHANICS

Abstract

A nuclear model of the hydrogen atom is devised consisting of N_h coplanar orbits each with a particle of mass nm and electronic charge $-e$ revolving in the nth orbit round a nucleus of charge $+N_h e$. A particle revolves through angle ψ in an unclosed elliptic orbit, at a distance r from the nucleus, as:

$$\frac{1}{r}=\frac{A}{n}\exp(-q\psi)\cos(\alpha\psi+\beta)+\frac{m\chi}{nL^2}$$

where A and β are determined from the initial conditions, q, α, and χ are constants, nL is a constant angular momentum in the nth orbit and m is the electronic mass. The number n (1, 2, 3...N_h) leads to quantisation of the orbits. The decay factor, $exp(-q\psi)$, is the result of radiation as a particle revolves and settles into the nth stable circle of radius $r_n = nL^2/m\chi$ with speed $v_n = \chi/nL$. It is shown that relative motions between the orbiting particles and the nucleus and between the particles themselves, give rise to radiation of discrete frequencies.

Keywords: Angular momentum, hydrogen atom, nucleus, orbit of revolution, radiation.

3.1 Introduction

Various models of the hydrogen atom have been introduced. The most successful of these models is the Rutherford's nuclear model introduced in 1911 [1].

3.1.1 Rutherford's nuclear model of the hydrogen atom

Lord Earnest Rutherford [1] proposed a nuclear theory of the atom consisting of a heavy positively charged central nucleus around which a cloud of negatively charged electrons revolve in circular orbits. The hydrogen atom is the simplest, consisting of one electron of charge $-e$ and mass m revolving in a circular orbit round a much heavier central nucleus of charge $+e$. This model, conceived on the basis of experimental results, has sufficed since, although with some difficulties regarding its stability and emitted radiation. This paper introduces a new nuclear model of the hydrogen atom for the liquid or solid state. The model is stabilized without recourse to Bohr's quantum mechanics.

According to classical electrodynamics [2] the electron of the Rutherford's model, in being accelerated towards the positively charged nucleus of the atom, by the centripetal force, should:

(i) emit radiation over a continuous range of frequencies with power proportional to the square of its acceleration and

(ii) lose potential energy and gain kinetic energy as it spirals into the nucleus, leading to collapse of the atom.

The second prediction is contradicted by observation as atoms are the most stable objects known in nature. The first effect is contradicted by experiments as a detailed study of the radiation from hydrogen gas, undertaken by J. J. Balmer as early as 1885, as described by Bitter [3], showed that the emitted radiation had discrete frequencies. The spectral lines in the Balmer series of the hydrogen spectrum satisfy the formula (Balmer formula):

$$\nu_{2q} = \frac{1}{\lambda_{2q}} = R\left(\frac{1}{2^2} - \frac{1}{q^2}\right) \tag{3.1}$$

where λ_{2q} is the wavelength, ν_{2q} is the wave number, R is the Rydberg constant and q is an integer greater than 2. The first of the four visible lines $(q = 3)$ is red. The spectral series limit $(q \rightarrow \infty)$, lying in the violet (not visible) region of the spectrum, is $\nu_2 = R/4$.

Niels Bohr [4] brilliantly rescued the atom from radiating and collapsing by invoking the quantum theory and making two postulates that prevented the atom from radiating energy. Otherwise the atom should emit radiation resulting in its collapse. Bohr's postulates are:

(i) In those (quantum) orbits where the angular momentum is $nh/2\pi$, n being an integer and h the Planck constant, the energy of the electron is constant.

(ii) The electron can pass from an orbit of total energy E_q to an inner orbit of total energy E_n in a quantum jump, the difference being released in the form of radiation of frequency f_{nq}, with energy given by $E_q - E_n = hf_{nq}$

The first postulate quantized the angular momentum with respect to (quantum) number *n*. With these postulates Bohr was able to derive a formula for the wave numbers of the lines of the spectrum of the hydrogen atom in exactly the same mathematical form as obtained by Balmer and generalized by J.R. Rydberg in 1889. Bohr's model gives:

$$\nu_{nq} = \frac{1}{\lambda_{nq}} = \frac{me^4}{8c\varepsilon_o^2 h^3}\left(\frac{1}{n^2} - \frac{1}{q^2}\right) \textit{per metre} \qquad (3.2)$$

$$\frac{1}{\lambda_{nq}} = R\left(\frac{1}{n^2} - \frac{1}{q^2}\right) \textit{ per metre} \qquad (3.3)$$

where *n* and *q* are integers greater than *0*, with *q* greater than *n*, *c* is the speed of light in a vacuum, ε_o is the permittivity of a vacuum and *h* is the Planck constant. Equation (3.3) is the Balmer-Rydberg formula giving the Rydberg constant *R* as:

$$R = \frac{me^4}{8c\varepsilon_o^2 h^3} \textit{ per metre} \qquad (3.4)$$

Substituting the values of the physical quantities in equation (3.4), *R* is found as 1.097×10^7 *per metre*, in agreement with observation.

In equation (3.3), if $n = 1$ we have the Lyman series, in the far ultra-violet region of the spectrum. *R* is the spectral limit for the Lyman series. For $n = 2$, we have the Balmer (4 visible lines) series and where $n = 3$ we get the Paschen series in the near infra-red region. In a series the lines crowd together and their intensities should decrease to zero as the series limit $(q \to \infty)$ is approached. Other series, in the infrared region, are obtained for $n = 4, 5, 6\ldots.$

The fact that a purely chance agreement between the quantities in equation (3.4) is highly improbable, lends plausibility to Bohr's theory of the hydrogen atom. This theory gave great impetus to quantum mechanics and was recognized as a remarkable triumph of the human intellect.

However, the quantum jump and absence of a direct link between the frequency of the emitted radiation and the frequency of revolution of the electron, in its orbit, leaves a question mark on Bohr's quantum theory.

Subsequently, Bohr's quantum theory was modified, notably by Sommerfield [5], for elliptic orbits and more complex atoms. This imposed extra ad-hoc conditions justifiable only by the (not always) correctness of their consequences.

3.1.2 An alternative nuclear model of the hydrogen atom

An alternative nuclear model of the hydrogen atom is introduced in this paper. The new nuclear model is for the solid or liquid state of the hydrogen atom. It is inherently stable outside quantum mechanics.

The new nuclear model consists of N_h coplanar orbits. A particle of charge $-e$ and mass nm revolves in the nth circular orbit with speed v_1/n and angular momentum nL at a distance nr_1 from a nucleus of charge $+N_he$. The radius r_1, speed v_1 and angular momentum L are with respect to the first orbit, the innermost orbit where $n = 1$.

In the stable state, a particle of mass nm revolves in the nth circular orbit. If a particle is disturbed from the circular orbit, it revolves as a radiator emitting a burst of radiation of increasing frequency and decreasing intensity as it spirals out towards the stable circular orbit. The frequencies of emitted radiation are very nearly equal to that of revolution of the particle in the nth stable circular orbit.

3.2 New nuclear model of the hydrogen atom

The new nuclear model of the hydrogen atom consists of a concentric arrangement of N_h coplanar orbits. The orbits are equally spaced, each with a particle of charge $-e$ and mass nm revolving in a circle round a nucleus of charge $+N_he$.

3.2.1 Equation of the orbit of motion

The equation of motion of a particle of mass nm revolving, in the nth orbit, with constant angular momentum nL, at a point distance r from the nucleus, is derived in equation (2.18) [6] as:

$$\frac{1}{r} = \frac{A}{n}\exp(-q\psi)\cos(\alpha\psi + \beta) + \frac{m\chi}{nL^2} \qquad (3.5)$$

where the amplitude of the excitement A and phase angle β are determined from the initial conditions and q, α and χ are constants.

The exponential decay factor $(-q\psi)$, in equation (3.5), is as a result of radiation of energy. The negatively charged particle will revolve, round the positively charged nucleus, in an unclosed (aperiodic) elliptic orbit with many cycles of revolutions, radiating energy at the frequency of revolution, before settling down into the stable circle of radius $nL^2/m\chi$, until it is disturbed again.

3.2.2 Radiation from the new nuclear model

Figure 3.1 represents the new nuclear model of the hydrogen atom. It consists of a concentric arrangement of N_h coplanar orbits. The orbits are equally spaced, each with a particle of charge $-e$ and mass nm revolving round a nucleus of charge $+N_he$, n being an integer: $1, 2, 3, 4.....N_h$. A circular orbit has radius $r_n = nr_1 = nL^2/m\chi$ in which a particle revolves with speed $v_n = v_1/n = \chi/nL$, where $\chi = N_he^2/4\pi\varepsilon_o$ and $n = 1$ for the innermost (first) orbit.

The frequency of revolution of a particle in the nth stable orbit, a circle of radius r_n, is:

$$f_n = \frac{v_n}{2\pi r_n} = \frac{1}{2\pi}\frac{\chi}{nL}\frac{m\chi}{nL^2} = \frac{m\chi^2}{2\pi n^2 L^3} = \frac{mN_h^2 e^4}{2\pi\left(4\pi\varepsilon_o\right)^2 L^3}\frac{1}{n^2}$$

$$f_n = \frac{mN_h^2 e^4}{2\pi\left(4\pi\varepsilon_o\right)^2 L^3}\frac{1}{n^2} = \frac{cS}{n^2} \tag{3.6}$$

$$S = \frac{mN_h^2 e^4}{2\pi c\left(4\pi\varepsilon_o\right)^2 L^3} \tag{3.7}$$

where S is a constant. If an electron is disturbed or dislodged from the stable circular orbit, it revolves in an unclosed elliptic orbit, emitting a burst of radiation in a narrow band of frequencies nearly equal to the frequency f_n of the circular revolution (equation 3.6).

Let us now follow the motion of two particles at positions P and Q with radii nr_1 and qr_1 respectively, revolving in anticlockwise sense round

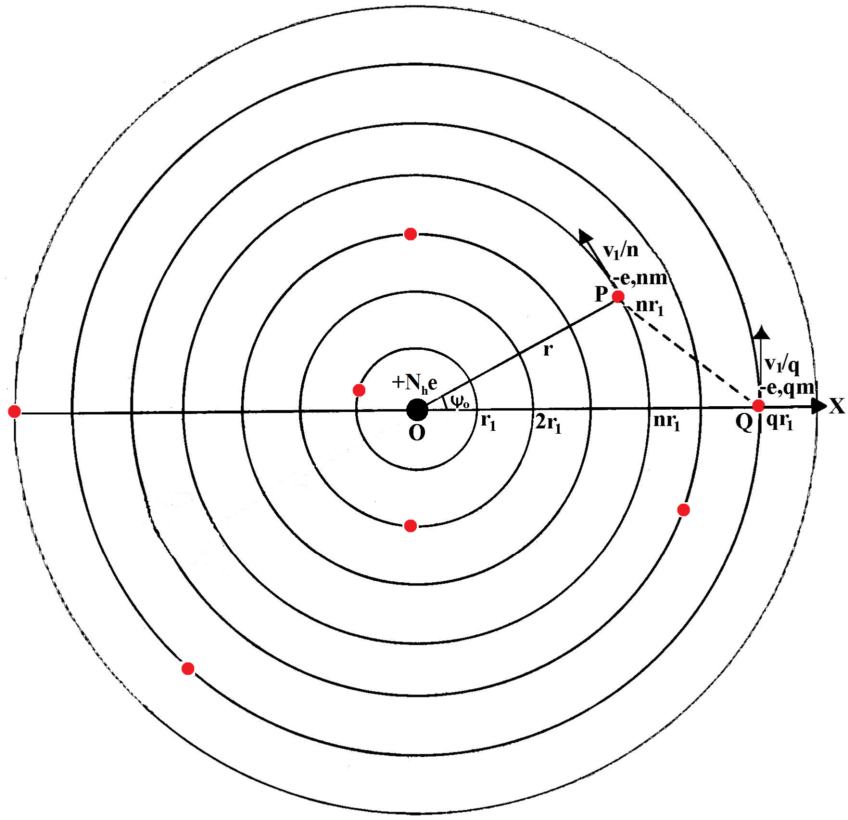

Figure 3.1 New nuclear model of the hydrogen atom, consisting of N_h coplanar orbits each with a negatively charged particle revolving anticlockwise, in angle ψ, under the attraction of a nucleus of charge $+N_he$. A particle in the nth orbit has a multiple of the electronic mass *nm* and charge $-e$, *n* being an integer. The particle in the nth orbit revolves in a circle of radius nr_1 with velocity v_1/n and angular momentum $nmv_1r_1 = nL$

the centre *O* as in Figure 3.1. The frequencies of revolution at *P* and *Q* are given by equation (3.6) for the respective orbital numbers *n* and *q*.

In Figure 3.1, let the particles at positions *P* and *Q* be as shown at the initial time $t = 0$. The relative positions of the points *O, P* and *Q* are as shown, with ***OP*** and ***OQ*** in an angular displacement ψ_o at the initial stage. In time *t* the line ***OP*** moves to $\boldsymbol{OP_t}$ through an angle ψ_n and line ***OQ*** moves to $\boldsymbol{OQ_t}$ through an angle ψ_q. The difference in angular displacement, the instantaneous angle $\boldsymbol{P_tOQ_t}$, is:

$$\psi_t = \psi_o + \psi_n - \psi_q$$

The angular frequency of oscillation of the particles at *P* and *Q*, is:

$$\frac{d\psi_t}{dt} = \frac{d\psi_n}{dt} - \frac{d\psi_q}{dt} = \omega_n - \omega_q = 2\pi f_n - 2\pi f_q = 2\pi f_{nq} \qquad (3.8)$$

Combining equation (3.8) above with equation (3.6) where $f_n = cS/n^2$ and $f_q = cS/q^2$, gives:

$$f_{nq} = cS\left(\frac{1}{n^2} - \frac{1}{q^2}\right) \qquad (3.9)$$

The wave number is:

$$\frac{f_{nq}}{c} = \frac{1}{\lambda_{nq}} = S\left(\frac{1}{n^2} - \frac{1}{q^2}\right) \textit{ per metre} \qquad (3.10)$$

Radiation from the new nuclear model of the hydrogen atom is of two sources. The first source is interaction between the electron revolving in the nth unstable orbit, and the nucleus, producing radiation of frequency f_n given by equation (3.6). The second source is interaction between electrons revolving in the nth and qth orbits, producing radiation of frequency f_{nq} given by equation (3.9), with the limit $(q \rightarrow \infty)$ given by equation (3.6).

3.2.3 Number of orbits in the new nuclear model

Measurements with a mass spectrometer showed that the hydrogen atom is about 1836 times the mass *m* of the electron. The total number N_h of orbits, each containing one particle of mass *nm* in the hydrogen atom, is obtained from the sum of the natural numbers: $n = 1, 2, 3, \ldots N_h$. This sum, which carries half of the mass of the atom, comes to: $N_h(N_h + 1)m/2 = 1836m/2$. Solving the quadratic equation gives $N_h = 42.35$. Obviously, N_h should be an integer, *42* or *43* or some other number. Further investigation and experimental work is required here to ascertain the actual number N_h of orbits.

3.3 Conclusion

The particles, in two orbits, emit radiation in a narrow band of frequencies and wave numbers as given by equations (3.9) and (3.10) respectively. At the same time, there is radiation of frequency given by equation (3.6) as a result of interaction between the orbiting particles and the nucleus. This is what this paper has set out to derive without recourse to quantum mechanics. In the process, the frequencies of emitted radiation are directly related to the frequencies of revolutions of the charged particles; something which quantum mechanics failed to do.

Equation (3.10) is similar to the Balmer-Rydberg formula (equation 3.3), but the Rydberg constant *R* (equation 3.4) is different from *S* (equation 3.7). The Balmer-Rydberg formula for the emitted radiation from the hydrogen atom, being the result of experimental observations, must stand for something. It is suggested here that the new nuclear model obtains with the liquid or solid phase of hydrogen.

The new nuclear model of the hydrogen atom is like the solar system with the planets revolving round a much heavier centre of attraction (the Sun). Should the centre fall apart, the particles of the nuclear model (in the solid or liquid state) would disperse to become a non-nuclear model (in the gaseous state). This issue is treated in paper 4 [8].

3.4 References

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