10 MICHELSON-MORLEY, SAGNAC AND MICHELSON-GAIL-PEARSON EXPERIMENTS

Abstract

The Michelson-Morley (MM) experiment recorded the famous null result. The Sagnac and Michelson-Gale-Pearson (MGP) experiments were conducted to observe fringe shifts that could occur as a result of interference between two light rays from the same source, reflected off mirrors, taking different times to move along the same path but in opposite directions. The Sagnac experiment gave a positive result and the MGP experiment also produced a positive result, which were regarded as evidences of the effect of rotational motion on the speed of light. In this paper it is shown that the result of the MM experiment should not be a null but a second order fringe shift which was not reckoned with. It is also pointed out that the formula used for calculating the fringe shift in the MGP experiment is not definite.

Keywords: Fringe shift, light, reflection, rotation, time, velocity

10.1 Introduction

In this paper the velocity of light $\boldsymbol{z}$, from a source moving with velocity $\boldsymbol{u}$, relative to an observer moving with velocity $\boldsymbol{v}$, is proposed as vector:

$$\boldsymbol{z} = \boldsymbol{c} + (\boldsymbol{u} - \boldsymbol{v}) \tag{10.1}$$

where $\boldsymbol{c}$ is the velocity of light relative to its source. For an observer at a point P moving with velocity $\boldsymbol{v}$ at an angle θ to a ray of light from a stationary source where $\boldsymbol{u} = 0$ relative to a measurement frame, as depicted in Figure 10.1, equation (10.1) becomes:

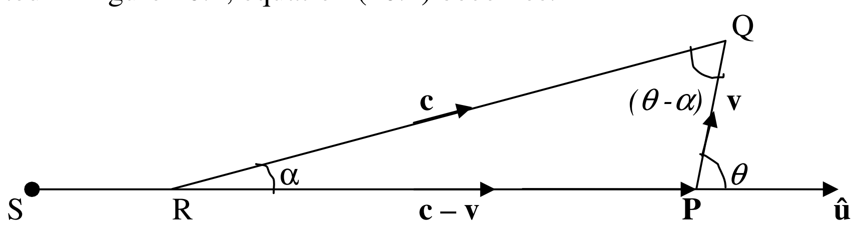

Fig. 10.1 An observer at P moving with velocity $\boldsymbol{v}$ at angle θ to light ray from source S

$$\mathbf{z} = \mathbf{c} - \mathbf{v} = \sqrt{c^2 + v^2 - 2cv\{\cos(\theta - \alpha)\}}\,\hat{\mathbf{u}} \tag{10.2}$$

where $\hat{\mathbf{u}}$ is a unit vector along the instataneou line joining S and P, in the direction of propagation of light. As a result of motion of the observer, the light ray appears displaced by aberration angle α such that:

$$\sin\alpha = \frac{v}{c}\sin\theta \tag{10.3}$$

For motion in the direction of propagation of light ($\theta = 0$), equations (10.2) and (10.3) give:

$$z = c - v \tag{10.4}$$

Motion against the direction of propagation ($\theta = \pi$ radians), gives:

$$z = c + v \tag{10.5}$$

For motion perpendicular to the direction of propagation of light ($\theta = \pi/2$ radians), equations (10.2) and (10.3) give:

$$z = \sqrt{c^2 - v^2} = c\sqrt{1 - \frac{v^2}{c^2}} \tag{10.6}$$

In the Michelson-Morley experiment [1, 2] the velocity of the observer (measurement frame) is along or perpendicular to the direction of light propagation. In this case equations (10.4), (10.5) and (10.6) apply.

In the Sagnac experiment [3, 4, 5, 6] equations (10.4) and (10.5) apply with speed v being the speed of rotation of a disc at radius L. This is also the case with the Michelson-Gale-Pearson (MGP) experiment [6, 7, 8, 9], where v is the speed of rotation of the Earth at a given latitude.

The MGP experiment was conducted in 1925 in a clearing at Chicago, Illinois, USA. It was repeated with greater precision, using laser, in 1995 [6] in New Zealand, Southern Hemisphere. In this experiment, there was a positive result attributed to the difference in transit times of two different rays of light moving in the opposite directions along coincidental paths. The two rays created a fringe shift δ observable in a telescope. The experiment took 269 separate readings and obtained values of δ ranging from –0.04 to +0.55 of a fringe with a mean of +0.26 fringes [10]. As for MM experiment, a small persistent positive shift has been reported [10].

The purpose of this paper is to show that the result of MM experiment is not a null but a small fringe shft. It is also to point out that the mathematical derivation of formula for the fringe shift, in terms of the wavelength of light used and angular frequency of rotation of the Earth at a latitude, which the experiment verified, was not definite.

10.2 Michelson-Morley experiment

A simplified diagram of the famous Michelson-Morley experiment is shown in Figure 10.2 below. The apparatus consisted of a light source *S*, a half-silvered mirror *A*, placed at an angle of 45^o, two mirrors *B* and *C* and a detector *D*. The whole apparatus, including the distant mirrors, was placed on a large turntablewhich that could be swung around by 90^o.

Light was directed at an angle of 45^o to a half-silvered, half transparent glass plate *A*, so that half of the light went on through the glass and half of it was reflected. The transmitted light went on to mirror *B* and the reflected beam to mirror *C*, each through an equal distance *L*.

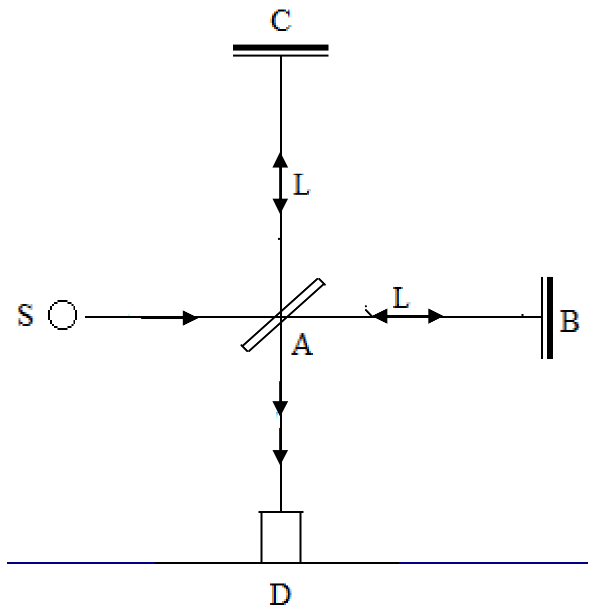

Figure 10.2 The apparatus of the Michelson-Morley Experiment

The reflected lights, from *B* and *C*, were returned to the half-silvered plate *A* where they were again half reflected and half transmitted. The recombined rays of light traveled behind the half-silvered plate *A* to reach the detector *D*, where interference fringes were recorded. Swinging the apparatus through 90^o should give a reading at the opposite side of the zero (centre) point of the fringe pattern.

If there was any difference in the transit times of the two rays, going through equal distance *L*, as a result of difference in relative speeds between the rays and the mirrors, it should show as interference fringes in the detector (interferometer) *D*. The effect on speed through the luminiferous ether on one of the rays was compared by rotating the spectrometer through 90^o. Then, by making measurements six months apart, one added or subtracted the speed of the Earth (30 km/s) in its orbit around the sun. The interferometer was easily sensitive enough to detect this effect if present. However, the shift obtained was 0.00 plus or minus 0.01 fringes, indicating a null result within the limits of resolution of the interfrometer.

Now, let us compute the effect of the arm *AB* (Figure 10.2) moving in the direction of the light ray with speed *v* and the arm *AC* moving in the perpendicular diection with speed *v*. The transit time of light going from *A* to *B*, obtained from equation (10.4) and from *B* to *A*, from equation (10.5) is:

$$t_1 = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2cL}{c^2 - v^2} \approx \frac{2L}{c}\left(1 + \frac{v^2}{c^2}\right) \tag{10.7}$$

Transit time of light going from *A* to *C* and *C* to *A* is obtained from equation (10.6) as:

$$t_2 = \frac{2L}{c\sqrt{1 - \frac{v^2}{c^2}}} \approx \frac{2L}{c}\left(1 + \frac{v^2}{2c^2}\right) \tag{10.8}$$

Transit time difference Δt between the two orthogonal rays is:

$$t_1 - t_2 = \Delta t = \frac{2L}{c}\left(1 + \frac{v^2}{c^2}\right) - \frac{2L}{c}\left(1 + \frac{v^2}{2c^2}\right) = \frac{Lv^2}{c^3} \tag{10.9}$$

The fringe shift δ, for light of wavelength λ, is:

$$\delta = c\frac{\Delta t}{\lambda} = \frac{Lv^2}{\lambda c^2} \tag{10.10}$$

10.3 Sagnac experiment

The apparatus of the Sagnac experiment consisted of a glass plate *A* and three mirrors *B*, *C* and *D* installed along the periphery of a disc of radius *R* mounted on a platform which was rotated anticlockwise at angular speed ω, as shown in Figure 10.2 below. A beam of light from a source *S* was divided by the glass plate *A* into two rays by refraction and reflection. The whole apparatus, including the source of light S and telescope *T*, were rotated with angular speed ω. The refracted (transmitted) ray travelled anticlockwise, in the direction of rotation, and the reflected ray travelled clockwise in the opposite direction of rotation. The returning rays were then recombined by plate *A* and passed into the telescope for recording of interference fringes.

In Figure 10.2, let us first regard the system as stationary, i.e. ω = *0*. In this case, $v = 0$ and $z = c$ (equation 10.4). The time t_o taken by the two rays, for the round trips covering a distance *4L* at speed *c*, is equal to:

$$t_o = \frac{4L}{c} \tag{10.11}$$

Zero (central) reading was obtained with the disc stationary. With the disc rotating at angular speed ω, the peripherial speed of plate *A* is $v = \omega R$, relative to the centre of rotation. To a good approximation, this speed is subtracted (equation 10.4) or added (equation 10.5) to the ray going anticlockwise or clockwise, a local activity affecting the speed of light.

Speed of the ray going anticlockwise (rotation), relative to the centre of rotation, is approximately equal to $(c - \omega R)$ and speed of the ray going clockwise *is* $(c + \omega R)$. Here, it is approximated that the ray of light moves along the periphery of the disc to which the mirrors are attached. The accuracy of this approximation can be increased by having many more reflecting mirrors or the rays can be guided by an optic fibre cable along the periphery of the disc. The time t_1 taken by the ray to go anticlockwise in the round trip of length 4L, is:

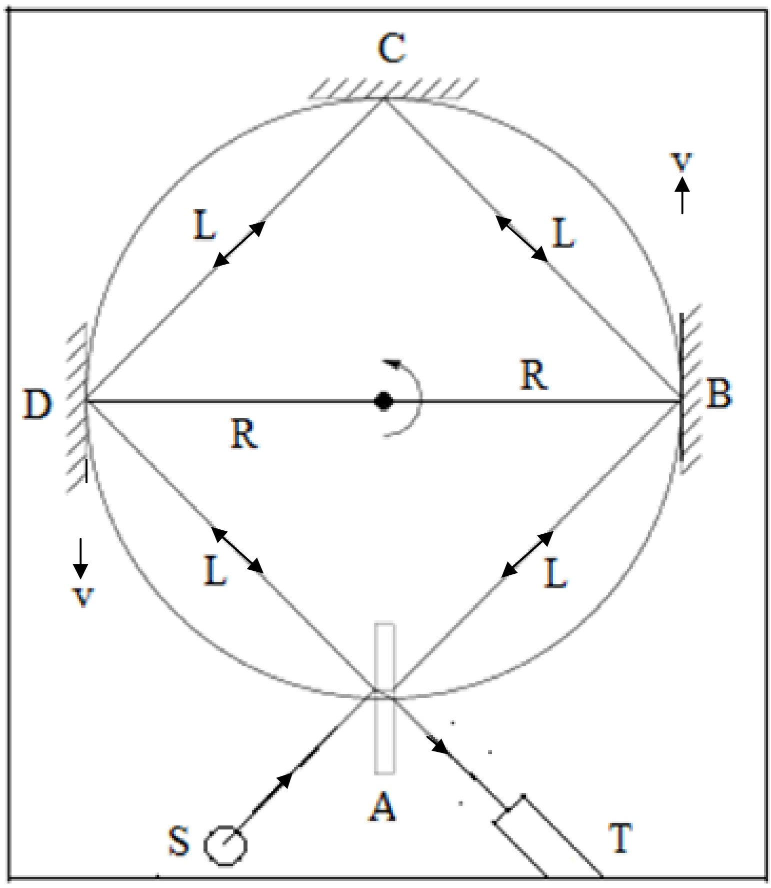

Figure 10.2 Apparatus of the Sagnac experiment

$$t_1 = \frac{4L}{c - \omega R} \qquad (10.12)$$

The time t_2 taken by the ray to go clockwise in the round trip of length 4L, is:

$$t_2 = \frac{4L}{c + \omega R} \qquad (10.13)$$

The time difference Δt between the two rays is:

$$t_1 - t_2 = \Delta t = 4L\left(\frac{1}{c - \omega R} - \frac{1}{c + \omega R}\right) \approx \frac{8\omega RL}{c^2} \approx \frac{4\omega\pi R^2}{c^2} \approx \frac{4\omega A}{c^2} \qquad (10.14)$$

where A is the area enclosed by the rotating mirrors. The fringe shift δ, for light of wavelength λ, is given by:

$$\delta = c\frac{\Delta t}{\lambda} = \frac{8\omega RL}{c\lambda} \qquad (10.15)$$

The *Sagnac Effect* or *Sagnac Interference* occurs in *Ring Interferometry* where two rays of light following a closed trajectory in opposite directions are made to combine and form an interference pattern. The Sagnac experiment has been performed a large number of times during the last century. Recently, using a ring laser [6] the Sagnac effect was confirmed to an extremely high degree of precision.

10.4 Michelson-Gale-Pearson experiment

Figure 10.3 is a schematic diagram of the arrangement and apparatus of the Michelson-Gale-Pearson (MGP) experiment [6, 7, 8]. The aim was to employ ring interferometry to measure the effect of speed of rotation of the Earth on the speed of propagation of light on the surface of the Earth.

As depicted in Figure 10.3, the MGP experiment consisted of a very large ring (rectangular) interferometer with a perimeter of about 1.9 kilometres, large enough to detect the angular velocity of rotation of the Earth. It was a modified version of the Michelson-Morley experiment with the mirrors tilted by 45^o so that the reflected rays of light are sent out to form a complete rectangle.

The rectangle consisted of 30-cm diameter water pipes *AEBCFD* about 612 metres by 339 metres, laid straight and level in a field, with the longer arms in the west-east direction and a smaller rectangle *AEFD* at one end. The pipes were evacuated using an air pump to make for clearer images. The larger rectangle carried a glass plate *A* and mirrors *B*, *C*, *D* at the corners and the smaller rectangle carried glass plates *A*, *E* and *F*. The plates were lightly coated to reflect and transmit the desired proportion of light and the mirrors were heavily silver-coated for full reflection.

A light beam from a carbon arc source *S* (Figure 10.3) was divided into two rays, a reflected ray and a transmitted ray, by plate *A*. The two rays of light were sent in opposite directions into the rectangle *AEBCFD*,

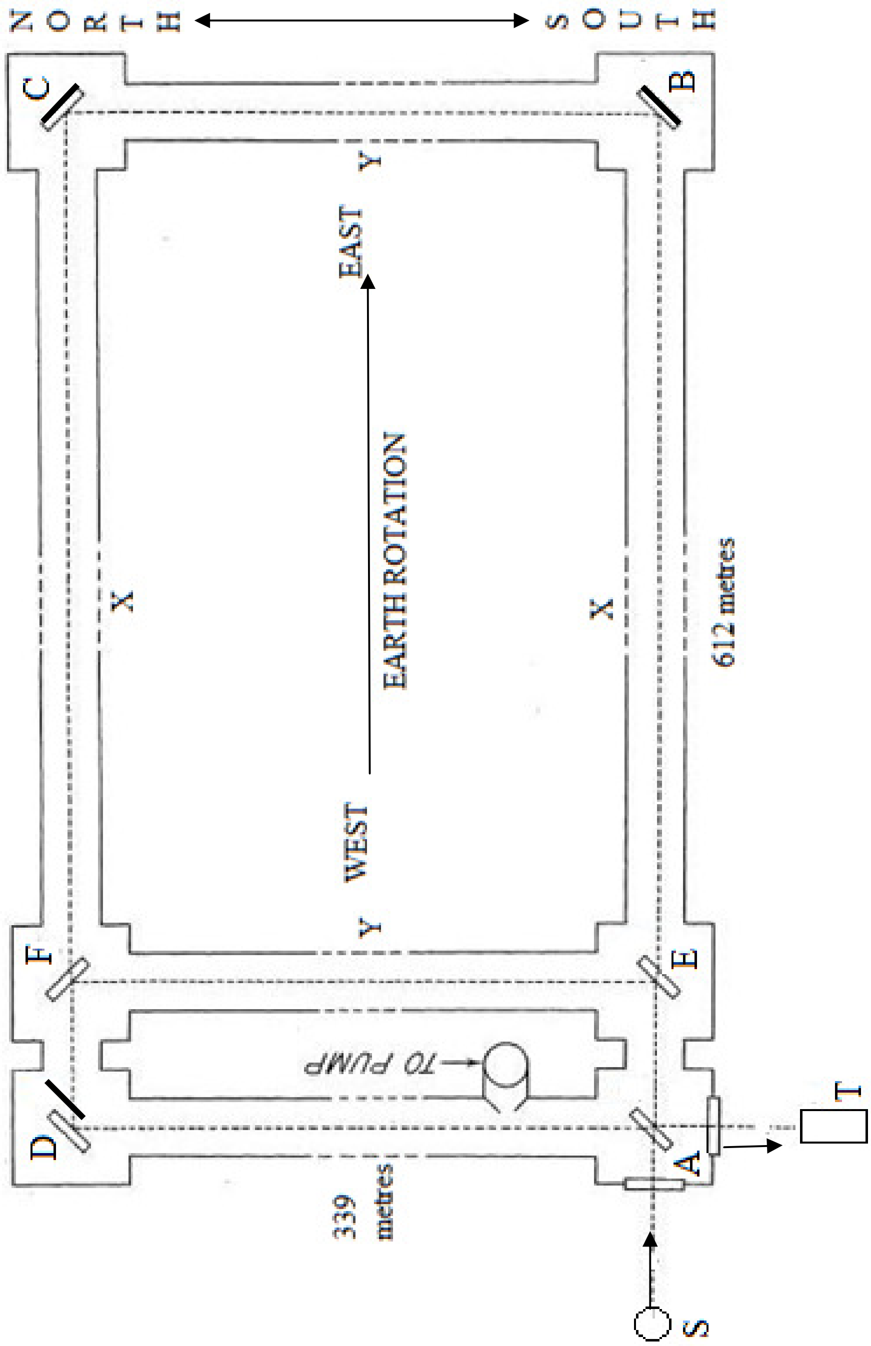

Figure 10.3 Apparatus of the Michelson–Gale–Pearson experiment

reflecting off the mirrors at the corners, and returned to the starting point *A*. Lights exiting the rectangle at *A* were compared in a telescope *T*. As a reading could not be taken with the Earth at a stand-still, the purpose of the smaller rectangle *AEFD*, with arms too short to provide any measurable observation, was to give a central (zero reference) point of the fringe pattern in the telescope..

In the absence of any effect due to the west-to-east rotation of the Earth, the two rays propagating in opposite directions in the rectangle *AEBCFD* of Figure 10.3, would go from and come back to the starting point *A* in an equal transit time. In this case the images would coincide at the centre of the screen of telescope *T*. As a result of rotation of the Earth, the arm *AEB* (length *X*) at latitude ϕ, being nearer the equator, would spin faster than the arm *DFC* (length *X*) at latitude $(\phi + \delta\phi)$, distance *Y* further north, This difference in speed of rotation would cause a difference in the speeds of light and a difference in transit times of the two rays going along the same path but in opposite directions; the ray going in the anti-clockwise direction *(AEBCFD)* taking a longer time in the round trip.

The speed of light in the arms *AEB* and *DFC*, each of length *X*, (Figure 10.3), relative to the rotating Earth, are found by subtracting or adding the speeds of rotation of the Earth along the respective arms. The determinations are as follows for the Earth of radius *R* at latitude ϕ, rotating at angular speed ω.

Speed of rotation along *AEB*, at latitude ϕ, is $v = \omega R\cos\phi$
speed of light going from *A* to *B* is $c - \omega R\cos\phi$
speed of light going from *B* to *A* is $c + \omega R\cos\phi$

Speed of rotation at *DFC*, latitude $(\phi + \delta\phi)$ is $v = \omega R\{\cos(\phi + \delta\phi)\}$
$= \omega R\{\cos\phi\cos(\delta\phi) - \sin\phi\sin(\delta\phi)\}$
$\approx \omega R\{\cos\phi(1 - Y^2/2R^2) - (Y/R)\sin\phi\}$
$\approx \omega\{R\cos\phi - (Y^2/2R)\cos\phi - Y\sin\phi\}$
where $\delta\phi \approx Y/R$, $\cos(\delta\phi) \approx 1 - Y^2/2R^2$ and $\sin(\delta\phi) \approx \delta\phi \approx Y/R$

speed of light going from *D* to *C* (distance *X*) *is*
$c - \omega\{R\cos\phi - (Y^2/2R)\cos\phi - Y\sin\phi\}$
speed of light going from *C* to *D* (distance *X*) *is*
$c + \omega\{R\cos\phi - (Y^2/2R)\cos\phi - Y\sin\phi\}$

Time taken for the round trip *ABCDA* is

$$t_1 = t_o + \frac{X}{c - \omega R \cos\phi} + \frac{X}{c + \omega\left(R\cos\phi - \frac{Y^2}{2R}\cos\phi - Ysi\,\text{n}\,\phi\right)}$$

$$t_1 \approx t_o + \frac{X}{c}\left(1 + \frac{\omega R \cos\phi}{c}\right) + \frac{X}{c}\left\{1 - \frac{\omega}{c}\left(R\cos\phi - \frac{Y^2}{2R}\cos\phi - Y\sin\phi\right)\right\}$$

$$t_1 \approx t_o + \frac{2X}{c} + \frac{\omega X}{c^2}\left(\frac{Y^2}{2R}\cos\phi + Ysi\,\text{n}\,\phi\right) \qquad (10.16)$$

where t_o is the time taken to cover the length *2Y* between the latitudes ϕ and $(\phi + \delta\phi)$.

Time taken for the round trip *ADCBA* is:

$$t_2 = t_o + \frac{X}{c + \omega R \cos\phi} + \frac{X}{c - \omega\left(R\cos\phi - \frac{Y^2}{2R}\cos\phi - Y\sin\phi\right)}$$

$$t_2 \approx t_o + \frac{X}{c}\left(1 - \frac{\omega R \cos\phi}{c}\right) + \frac{X}{c}\left\{1 + \frac{\omega}{c}\left(R\cos\phi - \frac{Y^2}{2R}\cos\phi - Y\sin\phi\right)\right\}$$

$$t_2 \approx t_o + \frac{2X}{c} - \frac{\omega X}{c^2}\left(\frac{Y^2}{2R}\cos\phi + Y\sin\phi\right) \qquad (10.17)$$

Time difference is

$$t_1 - t_2 = \Delta t = \frac{2\omega X}{c^2}\left(\frac{Y^2}{2R}\cos\phi + Y\sin\phi\right)$$

Fringe shift δ for light of wavelength λ is:

$$\delta = \frac{c\Delta t}{\lambda} = \frac{2\omega XY}{\lambda c}\left(\frac{Y}{2R}\cos\phi + \sin\phi\right) \approx \frac{2\omega XY\sin\phi}{\lambda c} = \frac{2\omega A\sin\phi}{\lambda c} \qquad (10.18)$$

where *A* is the area of the rectangle *XY* and *Y/R* is negligible. The fringe shift δ at the equator (ϕ = 0) is also negligible.

Equation (10.18) was originally obtained by Michelson in 1904 [2] but, in 1921 [3], the amount of shift was advisedly doubled to give:

$$\delta = \frac{4\omega XY \sin\phi}{\lambda c} \qquad (10.19)$$

The explanation for this doubling of fringe shift is not clear.

10.5 Conclusion

In MM experiment there should be a small fringe shift given by equation (10.10). For terrestrial speeds, even the speed of rotation of the Earth, this fringe shift is undetectable. For an MM interferometer, with arms 10 m long , the fringe shift due to speed of revolution of the Earth (30 km/s) is 0 to 0.175 for light of wavelength $\lambda = 5.7 \times 10^{-7}$ m, which is measurable.

The positive result of the Sagnac experiment is significant. The experiment was sufficient proof that the velocity of light can be added to or subtracted from in accordance with the classical law of addition of velocities but in contradistinction to the relativistic law.

In equation (10.10) for the MGP experiment, substituting for $\omega = 7.3 \times 10^{-5}$ rad/sec. X = 0.613 m, Y = 0.339 m, $\phi = 41° 46'$, $\lambda = 5.7 \times 10^{-7}$ m, c = 3×10^{8} m/s, the predicted fringe shift was computed as 0.237. The measured shift was found as 0.260, very close to prediction. This showed evidence of effect of rotation of the Earth on the speed of light on the Earth. Alternatively, measuring the fringe shift, at a given latitude, should determine the speed of rotation of the Earth (464 m/s at the equator). The MGP experiment should not be affected by orbital revolution of the Earth.

The use of equation (10) rather than equation (9), differing by a factor of 2, is an issue regarding the MGP experiment. The rays of light returning to plate *A* (Figure 3) and sent into the source may be reflected and returned back into the apparatus to cause a repetition of the round trips and give some spurious doubling of the fringe shifts in the telescope. It might as well have been that the rays of light inside the pipes of the MGP experiment were not moving along the same path.

The 30-cm pipes in the MGP experiment, laid flat and level on the earth’s surface should be arcs of circles (latitudes and longitudes), whereas light travels in a straight line. So the two rays of light would move in opposite directions in the pipes but not necessarily along coincidental paths. Any difference in circuit or path length should create a difference in transit times leading to an apparent fringe shift. The MGP experiment should be repeated with the 30-cm diameter pipe replaced with a narrower pipe, to make the two rays move in opposite directions nearly along the same path.

Indeed, there is no need for the water pipes in conducting the MGP experiment. The mirrors can be set up in a cleared field, at the corners of a rectangle, within distances of sight, such that the points of incidence are in one horizontal plane. The mirrors are then adjusted to make the incident and reflected rays lie in the same plane. At any position between the mirrors it can be ascertained whether the two light rays or laser beams moving in opposite directions are in the same path or not and adjustment made accordingly.

10.6 References

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