

## APPENDIX 2

*“All good things that exist are the fruits of originality”.*

John Stewart Mill (1806 – 1873)

### **BORH’S DERIVATION OF BALMER-RYDBERG FORMULA THROUGH QUANTUM MECHANICS**

Niels Borh (1885 – 1962), in a superbly original thought, derived the Balmer-Rydberg formula, for the spectral lines of radiation from the hydrogen atom, by invoking the quantum theory and making two postulates. The first postulate is that the electron, in the Rutherford nuclear model of the hydrogen atom, can revolve, without radiation, round the nucleus in allowed, quantum or stable orbits for which the angular momentum  $L_n$  is quantized, so that:

$$L_n = \frac{nh}{2\pi} \quad (\text{A2.1})$$

where  $n$ , the quantum number, is an integer greater than zero, and  $h$  is the Plank constant.

The second of Bohr’s postulate is that an excited electron translates from a stable orbit of radius  $r_q$ , corresponding to quantum number  $q$  and total energy (kinetic and potential)  $E_q$ , to an inner orbit of radius  $r_n$ , corresponding to quantum number  $n$  and total energy  $E_n$ . The electron loses potential energy and gains kinetic energy and, in the process, it emits radiation of frequency  $f_{nq}$ , in accordance with de Broglie’s hypothesis, such that:

$$E_q - E_n = hf_{nq} \quad (\text{A2.2})$$

where  $n$  is a number greater than zero but less than  $q$ .

Let us apply the two postulates of Bohr to the hydrogen atom whose electron of mass  $m$  and charge  $-e$  at a point  $P$  revolves with velocity  $v_n$  about a stationary nucleus of mass  $M$  and charge  $+e$  in a circular orbit of radius  $r_n$ , as shown in the Figure A2.1 below. The angular momentum of the electron, in the  $n$ th orbit, is

$$L_n = \frac{nh}{2\pi} = mv_n r_n \quad (\text{A2.3})$$

Equating the centripetal forces on the electron, we get::

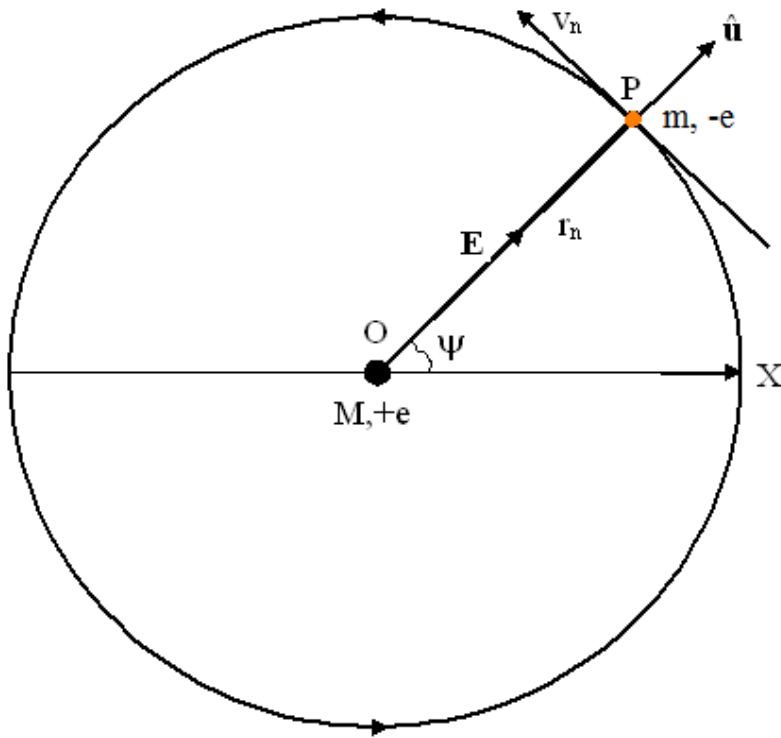


Figure A2.1 An electron of charge  $-e$  and mass  $m$  at a point  $P$  revolving with speed  $v_n$ , through an angle  $\psi$ , in a circle of radius  $r_n$  under the attraction of a heavy nucleus of mass  $M$  and charge  $+e$  at the central point  $O$ .

$$\frac{mv_n^2}{r_n} = \frac{e^2}{4\pi\epsilon_0 r_n^2} \quad (\text{A2.4})$$

Equations (A2.3) and (A2.4) give the speed and radius of revolution as:

$$v_n = \frac{e^2}{2\epsilon_0 nh} \quad (\text{A2.5})$$

$$r_n = \frac{\epsilon_0 n^2 h^2}{\pi m e^2} \quad (\text{A2.6})$$

With  $n = 1$ , we get the Bohr radius  $r_1 = 5.292 \times 10^{-7}$  m.

The total energy in the  $n$ th quantum state is obtained as:

$$E_n = \frac{1}{2}mv_n^2 - \frac{e^2}{4\pi\epsilon_0 r_n} \quad (\text{A2.7})$$

Substitute for  $v_n$  and  $r_n$  from equations (A2.5) and (A2.6) gives:

$$E_n = -\frac{me^4}{8\epsilon_0^2 n^2 h^2} \quad (\text{A2.8})$$

The total energy in the qth quantum state is:

$$E_q = -\frac{me^4}{8\epsilon_0^2 q^2 h^2} \quad (\text{A2.9})$$

Equation (A2.2) then becomes:

$$E_q - E_n = \frac{me^4}{8\epsilon_0^2 h^2} \left( \frac{1}{n^2} - \frac{1}{q^2} \right) = hf_{nq} \quad (\text{A2.10})$$

$$\frac{f_{nq}}{c} = \frac{1}{\lambda_{nq}} = \frac{me^4}{8c\epsilon_0^2 h^3} \left( \frac{1}{n^2} - \frac{1}{q^2} \right) \quad (\text{A2.11})$$

Equation (A2.11) is the Balmer-Rydberg formula (with  $n > 0 < q$ ) for the spectral lines of radiation from the hydrogen atom.

The stabilization of Rutherford's nuclear model of the hydrogen atom, by Bohr, was recognised as a brilliant achievement of the human mind. It gave an additional impetus to the development of quantum mechanics. However, the transition from one orbit to another, in zero time, as a necessary condition for radiation of energy, is a drawback on Bohr's quantum theory. So also is the failure to relate the frequency of emitted radiation to the frequency of revolution of the electron, round the positively charged nucleus.

Equation (A2.11) is an example of Bechmann's *Correspondence Theory*, whereby the expected, desired or correct result is obtained mathematically, but is based on wrong underlying principles. Wrong theories in learning and knowledge are like weeds in a farm. Unless good, fruit-bearing fruits are cultivated, weeds will take over. Once weeds have gained ground, it is very difficult to uproot them. But uprooted they must be if knowledge is to advance.